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# Conditional nonlinear optimal perturbation and its applications to the studies of weather and climate predictability

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**Abstract** **Conditional nonlinear optimal perturbation (CNOP) is the initial perturbation that has the largest nonlinear evolution at prediction time for initial perturbations satisfying certain physical constraint condition. It does not only represent the optimal precursor of certain weather or climate event, but also stand for the initial error which has largest effect on the prediction uncertainties at the prediction time. In sensitivity and stability analyses of fluid motion, CNOP also describes the most unstable (or most sensitive) mode. CNOP has been used to estimate the upper bound of the prediction error. These physical characteristics of CNOP are examined by applying respectively them to ENSO predictability studies and ocean's thermohaline circulation (THC) sensitivity analysis. In ENSO predictability studies, CNOP, rather than linear singular vector (LSV), represents the initial patterns that evolve into ENSO events most potentially, i.e. the optimal precursors for ENSO events. When initial perturbation is considered to be the initial error of ENSO, CNOP plays the role of the initial error that has largest effect on the prediction of ENSO. CNOP also derives the upper bound of prediction error of ENSO events. In the THC sensitivity and stability studies, by calculating the CNOP (most unstable perturbation) of THC, it is found that there is an asymmetric nonlinear response of ocean's THC to the finite amplitude perturbations. Finally, attention is paid to the feasibility of CNOP in more complicated model. It is shown that in a model with higher dimensions, CNOP can be computed successfully. The corresponding optimization algorithm is also shown to be efficient.**

**Keywords: nonlinearity, perturbation, predictability, nonlinear optimization.**

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One of the key problems in the studies of weather and climate predictability is the determination of the fastest-growing perturbation. To serve this purpose, Lorenz[1] introduced the concepts of linear singular vector (LSV) and linear singular value (LSVA), which were then widely used to find the fastest-growing perturbations of atmospheric flows. Recently, its applications have been extended to explore climate variability and predictability. Thompson[2] investigated the nonlinear dynamic behavior of LSVs of a coupled ENSO model and regarded them as the precursors of ENSO events. Ref. [3] employed LSV to study the error growth of a coupled ENSO model, in an attempt to explore the predictability of ENSO. Also Tziperman and Ioannou[4] applied this approach to finding a possible physical mechanism of transient amplification of initial perturbations to the thermohaline circulation. Obviously, LSV has become a widely used tool in the studies of weather and climate predictability.

Note that the linear theory of singular vector (SV) is established on the basis that the evolution of initial perturbation can be described approximately by the tangent linear model (TLM), which, due to the absence of nonlinearity, cannot describe the nonlinear evolution of the finite amplitude initial perturbations. Consequently LSV may not represent the fastest-growing perturbation of the nonlinear system. The motions of atmospheric and oceanic flows are generally nonlinear. Especially, El Niño-South- ern Oscillation (ENSO) shows itself highly irregularity and nonlinearity. The thermohaline circulation of ocean which has multiple equilibriums and internal oscillatory modes responds nonlinearly to the finite amplitude perturbation on a particular steady state. Even the simple two-dimensional quasi-geotropic model is also a nonlinear model. The nonlinearity limits the applications of LSV to these weather and climate systems.

Realizing the limitations of LSV, refs. [5, 6] modified the iterative procedure of the LSVs and LSVAs to construct the fastest-growing initial perturbations for the nonlinear regime. Recently, to study the effect of nonlinearity, Mu[7] employed nonlinear model directly and proposed a novel concept of nonlinear singular vector (NSV) and nonlinear singular value (NSVA). With a two-dimensional quasi-geotropic model, Mu and Wang[8] further studied the NSV and NSVA of the different basic flows. The results demonstrate that some types of basic flows have local fastest-growing perturbations, at which the objective function attains the local maximum. This phenomenon does not occur in LSV approach. The local fastest growing perturbations are usually of larger norms than the first NSV (nonlinear optimal perturbation). Although the growth rates of the local fastest growing perturbations are smaller than the first NSVA, their nonlinear evolution at the end of the time interval is considerably greater than that of the first NSV. In this case, the local fastest growing perturbations could play a more important role than the global fastest growing perturbation (first NSV) in the study of the predictability. Obviously, to study the predictability problem, we should first find out all local fastest growing perturbations, then investigate their impacts on the predictability. But this is inconvenient in application. Besides, the local fastest-growing perturbations with large amplitude of norm could be physically unreasonable.

To overcome the limitation of NSV and NSVA and to describe the initial perturbation that has the largest nonlinear evolution, ref. [9] introduced the concept of conditional nonlinear optimal perturbation (CNOP). CNOP is characterized by maximum nonlinear evolution of initial perturbations satisfying constraint conditions[9,10]. If the evolution of CNOP is measured by the growth rate of initial perturbation, CNOP may not be the fastest growing perturbation. But its nonlinear evolution describes the maximum evolution of initial perturbations at prediction time. Thus CNOP may play a more important role in the studies of predictability. In this sense, the authors regard this kind of initial perturbation as the constraint optimal perturbation of nonlinear system, i.e. CNOP. Mu and Duan[10,11] used CNOP to study the optimal precursor for ENSO and the effect of nonlinearity on error growth for ENSO. The CNOP was also used to investigate the nonlinear growth of instability of ocean's thermohaline circulation[12]. For a more realistic two-dimensional barotropic model, the CNOP was explored by Mu and Zhang[1)]. All these contributions to CNOP suggest that CNOP is a useful tool in predictability and stability or sensitivity studies. It is expected that CNOP can be widely applied in the studies of weather and climate predictability.

## 1 Conditional nonlinear optimal perturbation

We put the evolution equations for the state vector $w$ into an initial value problem:

$$\begin{cases} \dfrac{\partial w}{\partial t} + F(w) = 0, \\ w|_{t=0} = w_0, \end{cases} \quad \text{in} \quad \Omega \times [0,T], \tag{1}$$

where $w(x, t)=(w_1(x, t), w_2(x, t), \cdots, w_n(x, t))$, $x=(x_1, x_2 \cdots, x_n)$, $t$ is time, and $(x, t) \in \Omega \times [0, T]$, $\Omega$ a domain in $R^n$, $T< +\infty$. The operator $F$ in eq. (1) is a nonlinear differential operator, and $w_0$ is the initial state. Assume that the dynamical system equations and the initial state are known exactly, and the future state can be determined by integrating eq. (2) with the appropriate initial condition. The solution to eq. (1) for the state vector $w$ at time $t$ is given by

$$w(x, t)=M_t(w_0), \tag{2}$$

where, $M_t$ is the discrete propagator and stands for the numerical model. Let $U(x, t)$ and $U(x, t)+u(x, t)$ be the solutions of eq. (1) with initial value $U_0$ and $U_0+u_0$, respectively, where $u_0$ is an initial perturbation. We have

$$U(t)=M_t(U_0), \quad U(t)+u(t)=M_t(U_0+u_0). \tag{3}$$

So $u(t)$ describes the evolution of the initial perturbation $u_0$.

For a chosen norm $\|\cdot\|$, an initial perturbation $u_{0d}$ is called CNOP under the constraint $\|u_0\| \leqslant d$, if and only if

$$J(u_{0d}) = \max_{\|u_0\| \leqslant d} J(u_0),$$

where

$$J(u_0)=G(M_t(U_0+u_0)-M_t(U_0)), \tag{4}$$

where $G(\cdot)$ measures the evolution of the perturbation. Particularly it can be a norm ($\|\cdot\|$)of the state variables or the module ($|\cdot|$) of a variable. For the constraint condition, in this paper, it is simply expressed as belonging to a ball with the chosen norm. Obviously, we can also investigate the situation that the initial perturbations belong to another kind of functional set. Furthermore, the constraint condition could be some physical laws that initial perturbation should satisfy.

We emphasize that CNOP $u_{0d}$ is the global maximum of $J(u_0)$ in the ball $\|u_0\| \leqslant d$. It should be pointed out that the maximum value of $J(u_0)$ could be attained at several points in phase space. That is to say, the objective function may have several maximum. Another possibility is that there exists local maximum $u_{0d}^l$ of $J(u_0)$. In this case, we call $u_{0d}^l$ a local CNOP. CNOP and local CNOP can be computed by using sequential quadratic programming (SQP) solver[13], which is used to solve the nonlinear minimization problems with equality or/and inequality constraint condition. The detailed description of the algorithm is referred to ref. [14].

Now we turn to the physical meanings of CNOP. Firstly, if initial perturbations are expressed as initial anomalies of an anomaly model for climate, the corresponding CNOP, due to its optimality, plays the role of the optimal precursor of certain climate or weather event, which is most likely to develop into this weather or climate event[9,11]. Secondly, when CNOP is considered to be an initial perturbation superposed on a weather or climate event, for example, a realistic El Niño event, it acts as the initial error which has the largest effect on the uncertainty at the prediction time[9,10]. Thirdly, in the studies of sensitivity and stability analysis, since CNOP characterizes the maximum nonlinear evolution at prediction time for the initial perturbations satisfying the given constraint, it describes the most unstable (or most sensitive) initial perturbation of the nonlinear model with the given finite time period[12]. Finally, CNOP can also be used to estimate the upper bound of the prediction error. Assuming $U_0$ is an initial observation, and $U_T^t$ is the true value of the state, then the prediction error is

$$E = \left\| M_t\left(U_0\right) - U_T^t \right\|,$$

where $M_t$ represents the propagator of a forecast model. If the propagator $M_t$ is considered to be exact, the prediction error $E$ at prediction time $T$ is only caused by the initial

1) Mu Mu, Zhang Zhiyue, Conditional nonlinear optimal perturbation of a barotropic model, submitted to J. Atmos. Sci.

observational error. Since the true value of the state cannot be obtained exactly, it is impossible to get the exact value of $E$ (prediction error). But if we know some information on the errors of the initial observation, e.g., the initial observation error in terms of a norm is less than $d$, we can estimate the prediction error,

$$E_u = \max_{\|u_0\| \leqslant d} \left\| M_t\left(U_0 + u_0\right) - M_t\left(U_0\right) \right\|,$$

where $u_0$ is the initial perturbation superposed on initial observation $U_0$ and satisfies the constraint condition $\|u_0\| \leqslant d$. Obviously, the inequality $E \leqslant E_u$ holds. $E_u$ gives the upper bound of the prediction error[15], whose expression is the same as $J(u_{0d})$ in eq. (4) with $U_0$ being initial observation. Thus, CNOP gives the upper bound of the prediction error caused by initial uncertainties satisfying the constraint condition.

To explore these physical meanings CNOP bears, scientists have applied CNOP to the studies of ENSO predictability and THC sensitivity and stability analyses, attempting to show them in the concrete climate predictability studies.

## 2 Applications of CNOP to ENSO predictability

Recent studies have shown that it is of great significance for improving ENSO predictability to find out the precursors for ENSO events and to explore the mechanism of the initial error growth[2,12,16−18].

Some studies have attempted using the fastest growing perturbation to identify optimal growing initial patterns for ENSO, that is, the optimal precursors[2,19], and to study the initial error evolution of ENSO[3]. However, LSV is always associated with the sufficiently small initial perturbations. And its corresponding TLM cannot always describe the maximum growth of the initial perturbation in nonlinear models. Thus, it remains questionable as to whether the LSV can be used to investigate the predictability of ENSO when nonlinearity plays a large effect in ENSO model.

Considering this point, ref. [11] employed CNOP to study the optimal precursors for ENSO events, attempting to reveal the effect of nonlinearity on the optimal ENSO initial patterns. In the study, a simple coupled ocean-atmosphere model of ref. [20] (hereafter called WF96 model) is adopted. To describe the maximum evolution of SSTA at prediction time, the authors took the maximum value of the SSTA module as the objective function, which is used to yield CNOP. The authors first computed the CNOPs of annual cycle. The results demonstrate that for the annual cycle, regardless of what the initial time is, there exist CNOP and local CNOP. These CNOPs (local CNOPs) are all on the boundary of the constraint disc and have robustly the patterns of negative (positive) SSTA and positive (negative) thermocline depth anomaly qualitatively. Fig.1 illustrates the distribution of them. It is shown that for the same amplitude initial perturbation, when it is large, CNOP and local CNOP are respectively quite different from the LSVs under the condition that they are of the same amplitude of norm. And the LSVs, with the amplitude increasing, show themselves

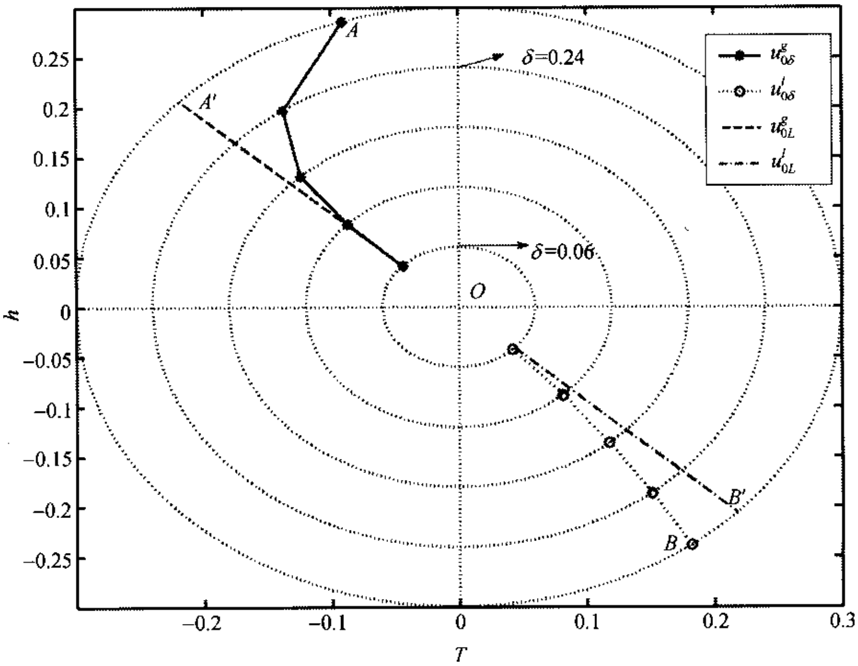


Fig. 1. The distribution of CNOPs (local CNOPs) and corresponding LSVs with an annual cycle in phase space, denoted by $u^g_{dL}(u^l_{0d})$ and $u^g_{0L}(u^l_{0L})$.

a beeline, while the CNOPs and local CNOPs shape into a curve respectively. Then the differences between CNOP (local CNOP) and LSV become more and more considerable with the increasing initial perturbation.

In ref. [11], the authors also investigated the evolution of CNOP (local CNOP) and LSV of annual cycle. The results show that for the short optimization time interval, no matter what the initial time is, there are only trivial differences between the linear and nonlinear evolution of the SSTA component of the CNOP (local CNOP), and between those of LSVs respectively. For the long time interval and large amplitude of initial perturbations, they have considerable differences respectively. Fig. 2 illustrates the differences between the linear and nonlinear evolution of CNOP (local CNOP). Besides, the differences of the nonlinear evolution of LSVs and CNOPs (local CNOP) were explored. For the same and large amplitude initial perturbation, the nonlinear SSTA evolution of CNOP is significantly larger than that of LSV (Fig. 2), which indicates that CNOP is optimal compared to LSV under the condition that they are of the same amplitude of norm. Further analysis demonstrates that the CNOP of annual cycle evolves into the positive sea surface temperature anomaly (SSTA) nonlinearly, which takes a striking resemblance to the development of El Ni o (Fig. 2). In fact, it acts as a precursor for El Niño event in WF96 model. Although the corresponding LSV also develops into an El Niño, the intensity is considerably weaker than that of CNOP (see Fig. 2). In this sense, they regarded CNOP as the optimal precursor for El Niño. For the local CNOP of annual cycle, its nonlinear SSTA evolution is only a little larger than that of the corresponding scaled LSV. This phenomenon can be explained by the locality of the local CNOP. As to the physical characteristic local CNOP bears, by investigating the nonlinear evolution, it is found that local CNOP acts as the optimal precursor of La Nina event in WF96 model.

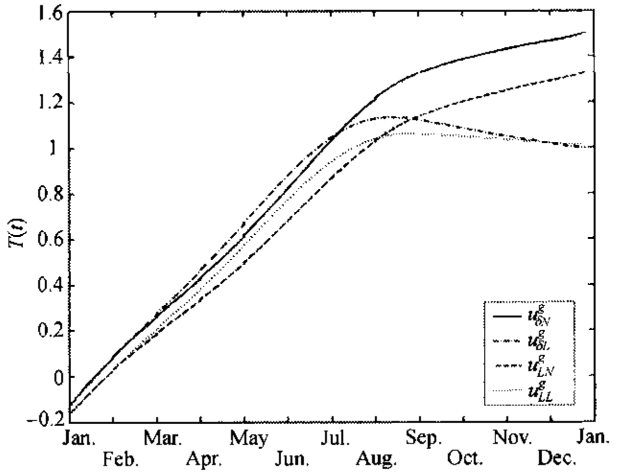


Fig. 2. Nonlinear and linear evolutions of the nondimensional model variable $T$(SSTA) corresponding to CNOP and LSV of annual cycle, respectively. $u^g_{dN}(u^g_{LN})$ and $u^g_{dL}(u^g_{LL})$ : the nonlinear and linear evolutions of SSTA of the CNOP (the LSV).

Finally, the authors compared the intensity of El Niño with that of La Nina (Fig. 3). They found that when they used LSV to study the intensity of ENSO events, the corresponding El Niño and La Nina events in TLM are of the equal amplitude, or say, the El Niño and La Nina events are symmetric about climatological mean state. While in CNOP approach, the El Niño event is obviously stronger than the La Nina event under the condition that the initial CNOP and local CNOP are of the same amplitude, which is quite consistent with the fact of the ocean data analysis. Clearly, the linear theory of singular vector cannot reveal the nonlinear asymmetry of El Niño and La Nina. The reason is that the El Niño-La Nina asymmetry is caused by a nonlinear feedback of WF96 model.

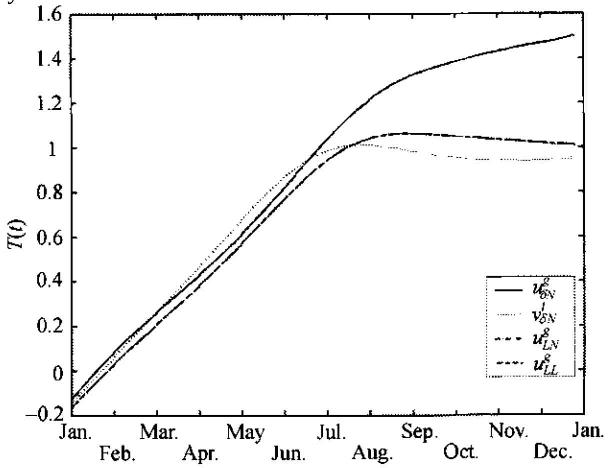


Fig. 3. Comparisons between El Niño and La Nina events. $u^g_{dN}(u^l_{LL})$: SSTA nonlinear (linear) evolution of CNOP (corresponding LSV), $v^l_{dN}(v^l_{LL})$ : SSTA nonlinear (linear) evolution of $-T$ (negative anomaly of SST) of local CNOP (corresponding LSV).

The above theoretical results are quite consistent with the 22-year NCEP reanalysis data qualitatively[11].

In the estimation of error growth, refs. [9, 10] used CNOP to investigate the error growth of ENSO events, especially to investigate the “spring predictability barrier (SPB)” problem of ENSO, where the SPB refers to the significant drop off in prediction skills for most of the ENSO models during the Northern Hemisphere springtime[21]. By computing the CNOPs of El Niño and La Nina events of the WF96 model, it is found that the error growth is enhanced for El Niño events and suppressed for La Nina events in spring. To further investigate what causes the SPB for ENSO in the model, the CNOPs of El Niño and La Nina events with strong and weak coupled ocean-atmosphere instability were also computed. The results suggest that the strong-coupled ocean-atmosphere instability during spring of the year is one of the causes of the SPB. Sensitivity experiments show that the SPB of

ENSO events has the tendency for phase-locking to the spring of the annual cycle.

In the application of CNOP to establishing the upper bound of prediction error, the authors chose the CNOP of annual cycle in WF96 model as the initial observation, which evolves into an ENSO event, to compute the CNOP (the optimal initial observation error). By investigating the evolution of the CNOP of the initial observation, the authors derived the upper bound of the prediction error of ENSO event. The results demonstrate that El Niño is less predictable than La Nina in the model. This result supports the one of ref. [19] and further emphasizes the effect of nonlinearity.

The above studies demonstrate the three physical characteristics of CNOP in predictability studies, i.e. the optimal precursor of certain climate event, the initial error which has largest effect on the predictability, and the one that establishes the upper bound of prediction error. All these emphasize that CNOP is superior to LSV in the study of predictability, and also more physically applicable than LSV.

### 3 The sensitivity and stability of the ocean's circulation to finite amplitude perturbations

The sensitivity of the ocean's THC is one of the fundamental issues on climate variability. Knutti and Stocker[22] investigated the sensitivity of the THC to perturbations. It is found that this sensitivity severely limits the predictability of the future THC when approaching the bifurcation point. Although LSV can be used to investigate the stability and sensitivity of the flow, in the sensitivity studies of THC, it cannot provide critical boundaries on finite amplitude stability of the thermohaline ocean circulation[12]. Furthermore, for a THC system with multiple equilibriums and internal oscillatory modes, as the introduction has mentioned, its response to a finite amplitude perturbation is a difficult nonlinear problem.

To reveal the effect of nonlinearity on the sensitivity of THC, ref. [12] employed CNOP to determine the nonlinear stability boundaries of linearly stable thermohaline flow states. With a simple two-box model of the thermohaline circulation[23], they also extended the results on linear optimal growth properties of perturbations on both thermal-and salinity-dominated thermohaline flows to the nonlinear case. It is shown that there is an asymmetric nonlinear response of these flows with respect to the sign of the finite amplitude freshwater perturbation.

In ref. [12], the authors computed the CNOPs of a two-box model of the thermohaline circulation respectively with thermal-dominated stable steady states (TH states) and salinity-dominated stable steady states (SA states), and studied the nonlinear developments of the finite amplitude perturbations of these two stable steady states for fixed model parameters.

In the case of TH states, the extensive numerical results were performed. It is demonstrated that the initial saline and freshwater perturbations of ocean's THC behave symmetrically with respect to the sign of steady flow rate in the corresponding TLM (Fig. 4). In the nonlinear two-box model adopted in ref. [12], due to the effect of nonlinearity, the nonlinear evolution of the freshwater (saline) perturbations leads to a larger (smaller) amplitude than their linear counterparts (Fig. 4). This indicates that the perturbations which move the system towards a bifurcation point will be more amplified through nonlinear mechanisms than perturbations that move the system away from a bifurcation point. The authors also demonstrate that for the CNOPs with small amplitude, the flow rate recovers to the steady climate state rapidly. For the CNOPs with large initial amplitude, it takes much longer for the thermohaline circulation to recover to steady state. This is different from the results of a linear analysis, which demonstrates that CNOP can reveal the effect of nonlinearity on THC.

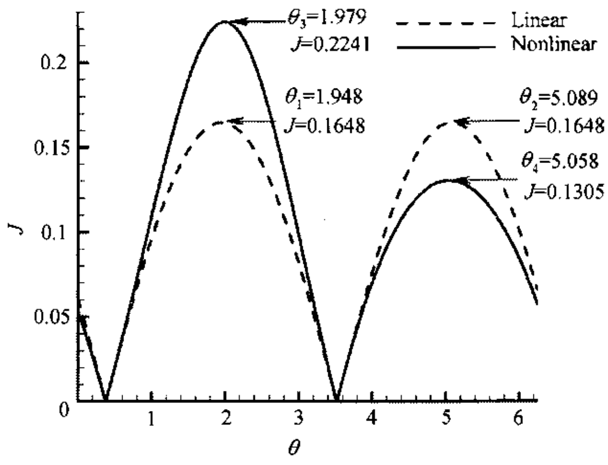


Fig. 4. The asymmetric nonlinear response of THC to initial freshwater perturbation. The solid line (dash line) represents the nonlinear (linear) evolution *J* of the initial perturbation. The nonlinearity makes the evolution of the freshwater perturbation much faster.

In the case of SA states, there are similar results to TH state, that is, the CNOP always moves the system towards the bifurcation point. The SA states have an asymmetry in the nonlinear amplification of disturbances, with larger amplitude for initial salinity perturbation.

In ref. [12], the authors also paid attention to the sensitivity of THC along the bifurcation diagram. The authors firstly calculated the CNOPs of the model along the TH branch for the continuous changing model parameters. The results demonstrate that with the parameter changing, the linearly stable TH state gradually transits from nonlinearly stable state to nonlinearly unstable state (Fig. 5). It is easily derived that for each value of the model parameter, a critical value of initial perturbation amplitude must exist such that the TH state is nonlinearly unstable, which induces a transition of the system from the TH state to the SA state (Fig. 6). This critical value acts as the nonlinearly stability threshold of the thermohaline flows. For the SA branch, there are similar results. For simplicity, the detailed description is not shown.

This work extends the CNOP approach to the field of sensitivity and nonlinear stability analyses. The results suggest that CNOP approach is a quite useful tool in sensitivity and nonlinear stability analyses and can be used to explore the effect of nonlinearity on the sensitivity and stability of perturbation. In the above THC sensitivity and stability studies, the CNOP represents the most unstable or the most sensitive initial perturbation of THC.

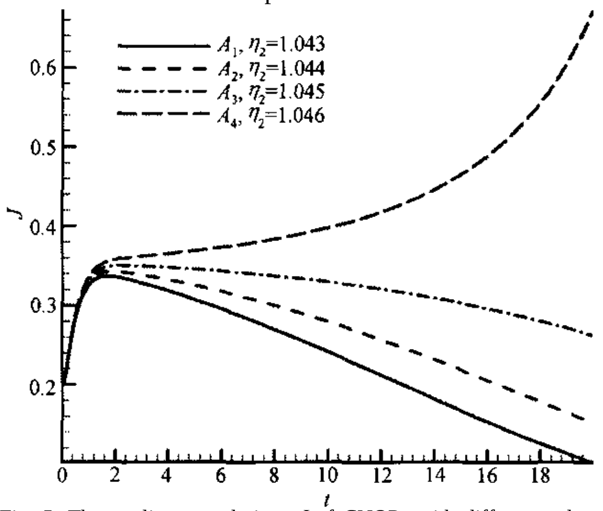


Fig. 5. The nonlinear evolutions $J$ of CNOPs with different values of freshwater parameter $h_2$. When $h_2$=1.046, the finite amplitude perturbation causes the transition of climate equilibrium.

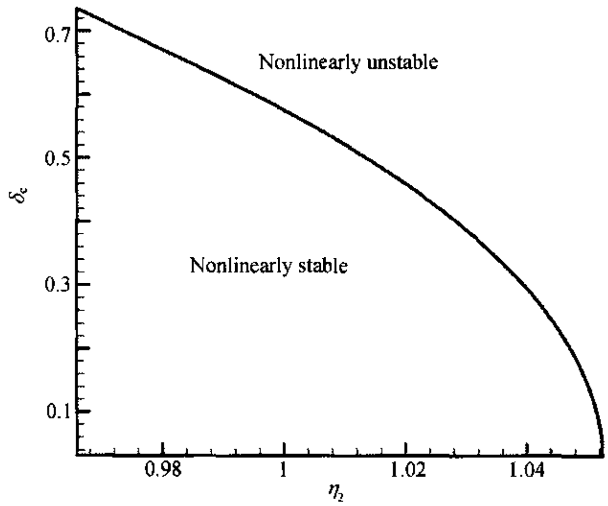


Fig. 6. The critical value $d_c$ (of initial perturbation amplitude) for nonlinear stability versus the parameter controlling the thermally-driven state near the saddle-node bifurcation at $h_2$=1.05.

1) See footnote 1) on page 2402.

## 4 Conditional nonlinear optimal perturbation of a barotropic model

Sections 3 and 4 have clarified the differences between CNOP and LSV by introducing the applications of CNOP to ENSO predictability and THC sensitivity analyses. All the work is for simple models of ordinary differential equations with two variables. It is necessary to investigate the feasibility of CNOP to a more general model of partial differential equations. Mu and Zhang1) have conducted extensive numerical experiments to study CNOP of a two dimensional quasi-geotropic model with dimensions 512 by using SQP algorithm. The results demonstrate the differences between CNOP and LSV from the two aspects of initial pattern and the evolutions. These two aspects emphasize that the significant differences between CNOP and LSV are aimed at the large initial perturbations and the long optimization time interval. Practically, these differences characterize the nonlinearity of the two-dimensional quasi-geotropic model, that is to say, CNOP reveals the effect of nonlinearity on perturbation growth.

All these above results suggest that CNOP approach is feasible in a realistic two dimensional barotropic model with dimensions 512. The SQP method is also shown to be successful. This encourages the application of CNOP to much higher dimensional problems.

## 5 Discussion

In this paper, CNOP and its applications are reviewed. Attention is firstly paid to the discussion of the physical meaning CNOP bears, then to the applications of CNOP to simple models and further to a higher-dimension model. The results demonstrate that CNOP can reveal the effect of nonlinearity on the predictability, sensitivity, and stability of weather and climate and shows itself the potential applicability in the predictability studies.

It is clear from these studies that in the applications of CNOP, the corresponding cost function and the constraint condition are of central importance, whose constructions should be capable of attacking the core of the physical problems that will be addressed. In this paper, to describe the evolution of the initial anomaly or the initial error, we choose the nonlinear evolution of the perturbation measured by the norm of the state variables or the module of a state variable as the cost function. The results show its effectiveness. As to the constraint condition, the authors simply express it as belonging to a ball with the chosen norm. Obviously, we can also investigate the situation that initial perturbations belong to another kind of functional set. Furthermore, the constraint condition could be some physical law that initial perturbations should satisfy.

Besides, in calculating CNOP numerically, an efficient nonlinear optimization algorithm is also essential, which guarantees the success of gaining CNOP. In this paper, for the low- and higher- dimensional model (maximal dimensions is 512), the sequential quadratic programme (SQP) algorithm has been shown to be successful for the nonlinearly constraint optimization problem. For the realistic forecast models, since they describe the intricate nonlinear atmospheric or oceanic flow motions and often have quite high dimensions, the involved nonlinear optimization problems could be difficult. Even in some cases, the problems are non-smooth. Nevertheless, encouraged by the work and the successful implemental of 4-dimensional variational data assimilations, it is expected that CNOP can be applied to more realistic models with quite high dimensions. Inspired by the applications of linear singular vectors (LSVs) to the ensemble forecast at European Center for Medium-Range Weather Forecasts (ECMWF), it is also worthwhile to investigate the applicability of CNOP in the ensemble forecast and to see if CNOP can serve the purpose of constructing the initial fields of ensemble forecast.

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# Conditional nonlinear optimal perturbation and its applications to the studies of weather and climate predictability

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