

Recent Advances in Predictability Studies in China (1999–2002)

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ABSTRACT

Since the last International Union of Geodesy and Geophysics (IUGG) General Assembly (1999), the predictability studies in China have made further progress during the period of 1999–2002. Firstly, three predictability sub-problems in numerical weather and climate prediction are classified, which are concerned with the maximum predictability time, the maximum prediction error, and the maximum allowable initial error, and then they are reduced into three nonlinear optimization problems. Secondly, the concepts of the nonlinear singular vector (NSV) and conditional nonlinear optimal perturbation (CNOP) are proposed, which have been utilized to study the predictability of numerical weather and climate prediction. The results suggest that the nonlinear characteristics of the motions of atmosphere and oceans can be revealed by NSV and CNOP. Thirdly, attention has also been paid to the relations between the predictability and spatial-temporal scale, and between the model predictability and the machine precision, of which the investigations disclose the importance of the spatial-temporal scale and machine precision in the study of predictability. Also the cell-to-cell mapping is adopted to analyze globally the predictability of climate, which could provide a new subject to the research workers. Furthermore, the predictability of the summer rainfall in China is investigated by using the method of correlation coefficients. The results demonstrate that the predictability of summer rainfall is different in different areas of China. Analysis of variance, which is one of the statistical methods applicable to the study of predictability, is also used to study the potential predictability of monthly mean temperature in China, of which the conclusion is that the monthly mean temperature over China is potentially predictable at a statistical significance level of 0.10. In addition, in the analysis of the predictability of the T106 objective analysis/forecasting field, the variance and the correlation coefficient are calculated to explore the distribution characteristics of the mean-square errors. Finally, the predictability of short-term climate prediction is investigated by using statistical methods or numerical simulation methods. It is demonstrated that the predictability of short-term climate in China depends not only on the region of China being investigated, but also on the time scale and the atmospheric internal dynamical process.

Key words: predictability, prediction, perturbation, computational uncertainty, weather, climate

1. Introduction

Predictability is a fundamental issue in both atmospheric and oceanic research and numerical weather and climate predictions. The terminology “predictability” is used extensively in the literature. However, in this review paper it is understood to refer mainly to the uncertainties of forecast results, which consists of two parts: the analysis of the factors and mechanisms that yield these uncertainties, and the search for methods and approaches to reduce these uncertainties. With more and more concern that people

have about the results of weather and climate prediction in daily life, the study of predictability is also becoming more and more important from both theoretical and practical views.

Presumably there are three approaches to exploring the predictability of atmosphere and ocean. The first is a statistical method, which calculates the variance of a time series or the correlation coefficients of the forecast results and observations; the second is related to the determination of the evolution of the difference between two “analogues” states from historical data (Lorenz, 1969); and the third approach consists

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of investigating the evolution of the initial errors by a numerical model, in which the “identical twin” control experiment (Lorenz, 1962, 1963; Kalnay, 2003) and linear singular vector (LSV) (Lorenz, 1965) are two important methods.

These above approaches have been widely applied to the study of predictability, and some successes have been achieved during the last few decades. The “identical twin” experiment of Lorenz (1962; 1963) discovered the fact that the atmosphere, like any dynamical system with instabilities, has a finite limit of predictability, which he estimates to be about two weeks. For the predictability problems of interannual climate such as ENSO (El Niño and Southern Oscillation), Chen et al. (1995) demonstrated the limit of ENSO’s predictability exceeds a year. Moore and Kleeman (1996), and Thompson (1998), etc., also utilized LSV to explore the rule of error growth with some coupled ocean-atmosphere models.

Chinese scientists have also made some significant contributions to the study of predictability of atmospheric and oceanic motions. The results of Yan et al. (1995) suggested that the forecast skill of monthly mean temperature could be improved by using the model corresponding to the fractal dimensions of a multi-level mapping model of the neural network back-propagation (BP) type. By using the IAP/CAS (Institute of Atmospheric Physics/Chinese Academy of Sciences) 9-Level AGCM, Wang et al. (1997) also supplied a possible way of increasing the predictability of the short-term climate, that is, improvement of land surface process modeling and the inclusion of the interannual variations of the land surface conditions (snow cover, albedo, soil moisture, etc.) as the forcing factors for climate modeling and prediction.

However, due to the great difficulties caused by the complexity and nonlinearity of the atmosphere and ocean, there are still many important problems that have not been investigated satisfactorily in the study of predictability, for example, the effect of nonlinearity on the predictability, and the dependence of numerical model predictability on computer precision, etc. In addition, with the development of numerical short-term climate prediction, it is important to study the predictability of seasonal climate prediction, the summer rainfall, etc.

Since the last International Union of Geodesy and Geophysics (IUGG) General Assembly (1999), four years have passed. During this period, Chinese scientists have further made some advances in the study of predictability. In the rest of this paper, we briefly introduce these works on the study of predictability during the period of 1999–2002.

2. Three predictability problems in numerical weather and climate prediction

Theoretically, the predictability problems are usually classified into two types according to the factors that cause the uncertainties of the forecast results. The first kind of predictability is related to the initial errors, and the second to the model errors. This classification is useful in theoretical study and in the improvement of numerical models. On the other hand, with the development of human society and economy, people require answers to the questions such as how large the prediction error is, and with given accuracy how long we can predict the weather or climate. With this in mind, Mu et al. (2002a, b) classified three predictability problems in numerical weather and climate prediction according to practical demands. The usefulness of this classification is that it provides an approach to quantitatively estimating the maximum predictability time, the maximum prediction error, and the allowed maximum initial error and parameter errors of the model. All of this information is important in the utilization of products of numerical weather and climate prediction. In the following, these three problems are formulated.

Problem 1. Assume that the initial observations and the first given values of the parameters of the model are known. At prediction time T , the prediction error in terms of a chosen measurement can be expressed by subtracting the true value of the state at time T from the prediction result. Our purpose is to gain the maximum predictable time for the given maximum allowed prediction error. Mathematically, this problem can be reduced to a nonlinear optimization problem. Since the true value of the state cannot be obtained exactly, this nonlinear optimization problem is unsolvable. But if we know more information about the errors of the initial values and parameters, a useful estimation of the maximum predictable time can be derived. Mu et al. (2002a) have established a lower bound on the maximum predictable time by investigating a corresponding solvable nonlinear optimization problem, which describes the lower bound of the maximum predictable time of the initial observation, and estimates the limit of predictability of the true state approximately.

Problem 2. Suppose that the initial observations and the first given values of the parameters of the model are known; then for a given prediction time, we look for the prediction error. Similar to the above problem, since the true value of the atmosphere cannot be known exactly, it is also impossible to get the exact value of the prediction error. But we can estimate it by using the information on the errors of the initial

observations and parameters. Mu et al. (2002a) investigated a nonlinear optimization problem, and proved that the solution of this nonlinear optimization problem yields an upper bound on the prediction errors.

Problem 3. Given the maximum allowed prediction error and the prediction time, for the initial observations and the first given parameters, we look for the maximum allowable initial error and the parameter error. Similar to the above two problems, Mu et al. (2002a) established a lower bound on the maximum allowable initial error and the parameter error by considering a corresponding nonlinear optimization problem.

It is worthwhile to point out that the above problems are formulated mainly according to numerical weather prediction, although the formulation can be applied to some climate predictions, for example, ENSO events. For numerical climate prediction, some necessary modifications concerning the choice of the norms and the objective functions are needed. But these will not bring essential difficulties.

Mu et al. (2002a) used the well-known Lorenz model to study the above three predictability problems by a numerical approach, which demonstrated how to realize the above ideas numerically.

3. The nonlinear singular vector (NSV) and nonlinear singular value (NSVA)

It is important to determine the fastest-growing initial perturbations in numerical weather and climate prediction and in atmospheric research. In the linear approach, to find the fastest-growing initial perturbation, it is assumed that the initial perturbation is sufficiently small such that its evolution can be governed approximately by the tangent linear model (TLM) of the nonlinear model. Lorenz (1965) introduced the LSV and linear singular value (LSVA) into meteorology to investigate the predictability of the atmospheric motion. Buizza and Palmer (1995) utilized LSVs to study the patterns of the atmospheric general circulation. Recently, this method has been used to find out the initial condition for optimal growth in coupled ocean-atmosphere models of ENSO, in an attempt to explore error growth and predictability of the coupled model (Xue et al., 1997a, b; Thompson, 1998; Samelson and Tziperman, 2001).

It is well known that the motions of the atmosphere and ocean are governed by complicated nonlinear systems. This raises a few questions concerning the validity of TLM. One is how small the initial perturbation should be to guarantee the validity of the TLM; another is how to determine the time interval during which the TLM is valid. There have been a few papers

to discuss these problems, and the results show that it is very difficult to determine the validity of the TLM in advance (Lacarra and Talagrand, 1988; Tanguay et al., 1995; Mu et al., 2000). Hence, for the nonlinear systems in numerical weather and climate prediction, it is desirable and often necessary to deal with the nonlinear models themselves rather than their linear approximations.

Mu (2000) proposed a novel concept of the NSVA and NSV, which is a natural generalization of the classical LSVA and LSV.

Mu and Wang (2001) used a two-dimensional quasi-geostrophic model to study the NSVA and NSV. The numerical results demonstrate that for some types of basic states, there exist local fastest-growing perturbations, which correspond to the local maximum values of the functional by which the NSVA and NSV are determined. But there is no such phenomenon in the case of LSVs and LSVA due to the absence of the nonlinearity of the corresponding TLM. The local fastest-growing perturbations usually possess larger norms compared to the first NSV, which corresponds to the global maximum value of the functional. Although the growth rates of the local fastest-growing perturbations are smaller than the first NSVA, their nonlinear evolutions at the end of the time interval are considerably greater than those of the first NSV in terms of the chosen norm. In this case, the local fastest-growing perturbations could play a more important role than the global fastest-growing perturbations in the study of the predictability.

It is clear from the results of Mu and Wang (2001) that, to study the predictability problem, we should first find out all local fastest-growing perturbations, then investigate their impacts on the predictability. But this is inconvenient in practical application. Besides, the local fastest-growing perturbation with a large norm could be physically unreasonable.

All these suggest that we should investigate the nonlinear optimal perturbation with constrained conditions.

4. Conditional nonlinear optimal perturbation and its applications to the predictability of climate

Mu et al. (2003) introduced the concept of conditional nonlinear optimal perturbation (CNOP), whose nonlinear evolution is maximal at the prediction time for the given constraint condition of the initial perturbations, to study the predictability of the ENSO (Mu and Duan, 2003; Mu et al., 2003; Duan, 2003). Firstly, Mu et al. (2003) and Duan (2003) employed

the approach of CNOP to find the precursors of the ENSO event within the frame of a simple coupled ocean-atmosphere model for ENSO. It is suggested that for the proper constraint condition, the CNOPs of the climatological mean state evolve into ENSO events more likely than the LSV. Consequently it is reasonable to regard CNOPs as the optimal precursors of ENSO events. An analysis of the observed anomalous monthly mean SST ($^{\circ}\text{C}$) and depth of the 20°C isotherm (m) derived from NCEP ocean reanalysis reveals that almost every ENSO event can be traced to the patterns that are similar to the precursors of ENSO events obtained by CNOP. This verifies the theoretical results of the optimal precursors of ENSO events.

Secondly, the “spring predictability barrier” problem for the ENSO event was also investigated (Mu and Duan, 2003; Duan, 2003), where the “spring predictability barrier” is a phenomenon for ENSO that the forecast skill falls rapidly during the spring of the year, regardless of when a forecast is started (Moore and Kleeman, 1996). By computing the CNOPs of El Niño and La Niña events, it is found that the error growth is enhanced in spring for El Niño events and suppressed in spring for La Niña events. To further investigate what causes the spring predictability barrier in the model, the CNOPs of El Niño and La Niña events with strong and weak coupled ocean-atmosphere instability are also computed. The results suggest that the strong-coupled ocean-atmosphere instability during the spring of the year is one of the causes of the spring predictability barrier. Sensitivity experiments show that the spring predictability barrier of ENSO events has the tendency for phase-locking to the spring of the annual cycle.

Besides this, CNOP was also used to analyze the different sensitivities of the thermohaline circulation to finite amplitude freshwater and salinity perturbations. Within the frame of a simple model for the thermohaline circulation, the impacts of nonlinearity on the evolution of the finite amplitude freshwater and salinity perturbations were investigated by the CNOP approach. It is demonstrated that the thermohaline circulation is more sensitive to the finite amplitude freshwater perturbation than the salinity perturbation. From the sensitivity analysis of the thermohaline circulation to the freshwater and salinity perturbations along the bifurcation diagram, it is shown that the system becomes unstable near the bifurcation diagram regime, and a finite perturbation can lead to the shut-off of the thermohaline circulation.

5. Monotonicity principle of predictability

The problem of predictability itself is essentially an

issue of the spatial-temporal scale (Chou, 2002; Mu et al., 2002b; Li and Chou, 2003a). Predictability time strongly depends on the spatial-temporal scale of the phenomenon studied. Moreover, it is also related to the initial condition and external forcing condition. Generally, the predictability time T_p of the atmosphere is a function of spatial and temporal scales, initial field and external forcing, i.e.,

$$T_p = T_p(D \times T; T_0, F), \quad (1)$$

where D and T are the space and time scales, respectively, X_0 is the initial field, and F the external forcing. This suggests that predictability of a system is determined by the four factors of its spatial scale, timescale, initial condition, and external forcing. By use of this function, Li and Chou (2003b) and Mu et al. (2002b) defined exactly the concepts of the stable component and chaotic component in the atmosphere. They pointed that under the same conditions of both initial field and external forcing, a larger spatial-temporal scale system possesses longer predictability time. This property, which is similar to Newton’s Law of Inertia that the bigger the mass, the bigger its inertia, is called the monotonicity principle of predictability. The principle shows that the model used to describe the sub-scale chaotic component (low level) of a system cannot be applied to predict the stable component (high level) of the system. To reduce blindness we should therefore focus on finding the system’s corresponding stable component for prediction of a specified spatial-temporal scale system (Li and Chou, 2003b).

6. Dependence of model predictability on machine precision

The traditional research on model predictability does not take machine precision into account. Based on the latest analysis, Li et al. (2000), Li (2000), and Li et al. (2001b) pointed out that this traditional manner is improper. On one hand, numerical methods themselves are not entirely accurate; on the other hand, any computer used to run models possesses finite precision. This implies that the model predictability time depends not only on the numerical scheme, but also on the machine precision as well as the model itself. Therefore, the model predictability time from the traditional research is neither the predictability time of the real atmosphere or climate nor the optimal predictability time of the model.

The results of Li et al. (2000) and Li et al. (2001b) indicated that round-off error due to finite machine precision has very significant influence in long-term numerical solutions of integration of the model, and this

finite precision in practice causes the computational uncertainty principle (Li et al., 2001b). This principle suggests that there is a limit to the ability of effective simulation by computer. This limit is inherent and is independent of the objects simulated (more precisely, except a zero measure set). The extent of the capacity of effective simulation, however, usually depends on the objects simulated. In light of the computational uncertainty principle, a new approach can be presented to study model predictability time; that is, through carrying out the optimal calculation of a numerical model, its maximum effective computational time (MECT) is obtained and then its predictability time in practice may be estimated. By use of the theory of the computational uncertainty principle, Li (2000) and Li et al. (2001b) presented an optimal numerical integration method of step-by-step adjustment, which can make the numerical model achieve the best predictability.

7. Global analysis on climate predictability

The cell-to-cell mapping method (Hsu, 1980, 1987; Chou, 1986) is a powerful tool for globally analyzing a nonlinear system. This method may be applied to quantitatively investigate the problem of predictability and to obtain the global predictability limit of a system for infinite initial conditions (Chou 1989). By introducing the cell-to-cell mapping method, Fan et al. (1999) studied the predictability of climate in a most simplified air-sea coupled model. Their results indicated that there exists a maximum predictability limit in climate prediction, and for the prediction beyond the daily predictability limit, the mean value is predictable. They also obtained and discussed some related quantitative results. Moreover, their studies implied that the coupling mechanism could prolong predictability time, and improvement of observational error might also extend the maximum predictability time.

8. Predictability of summer rainfall and monthly temperature in China

By use of 500-hPa geopotential height data and China's 160-station rainfall data, Zhu (1999) studied predictability of summer rainfall in China based on the relation between the rainfall pattern and the circulation in summer and the preceding winter. Her results showed that the characteristics of simultaneous and preceding circulations are significantly different among the different rainfall patterns, the predictability of summer rainfall is different in different areas of China, and there is stronger predictability in the lower

and middle reaches of the Yangtze River than other regions over East China.

Yue et al. (1999) investigated climatic noise and potential predictability of monthly mean temperature in China based on the analysis of China's 74-station temperature data. Their results implied that generally the climatic noise of monthly mean temperature in China increases with latitude and altitude and varies with the season, the potential predictability possesses strongly seasonal and regional dependences, and the monthly mean temperature over China is potentially predictable at the statistical significance level of 0.10. They further pointed out that for different seasons, a regional model could be a hopeful approach to predict the monthly mean temperature over China.

9. Predictability of T106 objective analysis/forecasting field

According to the need of daily weather prediction and the definition of the ambient field as the initial input of a regional forecasting model, Li et al. (2001a) preliminarily diagnosed the predictability of the objective analysis/forecasting field of the model with a triangular spectral truncation of 106 waves (T106 spectral model). Firstly, they investigated the distributions of the mean-square errors of the models: T106 spectral model, European Center for Medium Range Weather Forecasting (ECMWF) model, American National Meteorological Center Forecasting model (KWBC model), and Japan Global Spectral Model. Then, they further discussed the possibilities of the forecast of prediction error in the T106 itself. It is shown that the dominant error in T106 model is actually due to the fixed error in the T106 objective analysis field. Besides, an assembly objective analysis field is also presented to a weather forecaster to make a correct analysis, which may be the better initial-boundary condition as input being used in the regional forecasting model.

10. Predictability of short-term climate prediction

By using an ensemble of nine 17-year (1980–1996) hindcast experiments conducted with the first generation Atmospheric General Circulation Model (IAP L2 AGCM 1.1) of the Institute of Atmospheric Physics/Chinese Academy of Sciences, Zhao et al. (2000) applied the Analysis of Variance (ANOVA) technique to study the predictability of numerical short-term climate prediction. The results showed

that the predictability of atmospheric seasonal variations induced by SST anomalies is higher in the tropics and moderate in the extratropics except in some areas. In the extratropics, the predictability is higher in spring (March, April, and May) than in summer (June, July, and August). The predictability over the ocean is generally higher than over land. In China, especially, the predictability decreases from the South China Sea to the northwest for the fields of precipitation, sea level pressure, and air temperature.

Wang and Zhu (2000, 2001) reviewed studies on seasonal climate prediction, evaluated the level and skill of short-term climate prediction, and discussed the predictability of short-term climate forecasts. They pointed out that the scale score of seasonal prediction at present is 0.2–0.3 for temperature and 0.1–0.2 for precipitation, which correspond to an accuracy of 60%–65% and 55%–60%, and that the theoretical limit of monthly- and seasonal-scale climate prediction is about 6–12 months.

Employing the IAP 2-level AGCM and LASG 9-level spectral AGCM, Long and Li (2001) simulated the influence of positive SSTAs with different durations over the eastern equatorial Pacific on the subtropical high over the western Pacific. Their results showed that the predictability of the summer subtropical high over the western Pacific is determined by both the SSTAs over the equatorial eastern Pacific and the atmospheric internal dynamical process.

11. Summary

The progresses in the study of predictability in numerical weather and climate prediction achieved by Chinese scientists in the period of 1999–2002 are reviewed. The works consist of two parts: theoretical and practical investigations. In the former, three subproblems of predictability are classified and then are reduced into three nonlinear optimization problems. In order to describe the effects of nonlinearity on the evolution of error growth, the bases of the LSV, NSV and CNOP are also introduced, whose applications to the study of weather and climate predictability suggest that the nonlinearity of the motions can be disclosed by them. Attention is also paid to the investigation of the impacts of the spatial-temporal scale and machine precision on predictability. The results suggest that this investigation is helpful to improve the forecast skill and to realize the best predictability of the atmosphere and ocean. Besides these, the cell-to-cell mapping method is adopted to analyze globally the predictability of climate. Some significant results demonstrate that the coupling mechanism and the improvement of initial observational error can extend the

maximum predictability time and improve the predictability of climate. Therefore the theory of global analysis supplies a new subject to the predictability study of climate.

Concerning practical investigation, Chinese scientists employed the IAP 2-level AGCM, the T106 spectral model, and the LASG 9-level spectral AGCM to study the predictability of summer rainfall in China, monthly mean temperature in China, and numerical weather and short-term climate prediction. It is demonstrated that the predictability of summer rainfall in China is different in different areas of China and has stronger predictability in the lower and middle reaches of the Yangtze River than other regions over East China. For the prediction of the monthly mean temperature in China, it is proved to be helpful for the improvement of the predictability to employ a regional model for different seasons.

Predictability study is a field of challenge due to the nonlinearity and complexity of atmospheric and oceanic motions, however it is expected that, with the development of society and the improvement of the cognitive ability of people, significant progress in the study of predictability will be made in the future.

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