Applications of nonlinear optimization methods to quantifying the predictability of a numerical model for El Nino-Southern Oscillation*

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Abstract The nonlinear optimization methods are applied to quantify the predictability of a numerical model for El Nino-Southern Oscillation (ENSO). We establish a lower bound of maximum predictability time for the model ENSO events (i.e. ENSO events in the numerical model), an upper bound of maximum prediction error, and a lower bound of maximum allowable initial error, all of which potentially quantify the predictability of model ENSO. Numerical results reveal the phenomenon of "spring predictability barrier" (SPB) for ENSO event and support the previous views on SPB. Additionally, we also explore the differences between the linear evolution of prediction error and its nonlinear counterpart. The results demonstrate the limitation of linear estimation of prediction error. All these above results suggest that the nonlinear optimization method is one of the useful tools of quantifying the predictability of the numerical model for ENSO.

Keywords: ENSO model, nonlinear, optimization, predictability.

Since Lorenz[1] suggested that chaotic dynamics may set bounds on the predictability of weather and climate, assessing the predictability of various processes in the atmosphere-ocean system has been the objective of numerous studies. These studies are generally classified as the first kind and the second kind of predictability studies[2]. The former addresses how the initial uncertainties affect the prediction results, and the latter discusses mainly the effect of model error on the prediction uncertainties.

These two types of predictability studies play a dominant role in improving the forecast skill of numerical weather and climate prediction[3]. However, with the development of human society and economy, people require to know the answers to questions such as how long we can predict the weather and climate with a predetermined accuracy, and with a given prediction time, how large the prediction error is, and so on[4]. With this in mind, Mu et al.[5], considering the collective effects of initial error and model error on prediction results, classified the predictability of numerical weather and climate prediction into three predictability problems, which were then applied to quantify the predictability of Lorenz' model[1]. The results suggest that this classification of predictability problems provides a useful tool for estimating quantitatively the maximum predictable time of a model, the maximum prediction error and the maximum allowable initial errors.

El Nino-Southern Oscillation (ENSO) shows chaotic behaviors, the irregularity of which seriously limits the predictability of ENSO[6]. To improve the predictability of ENSO, during the last 20 years or so, scientists performed many researches[7—9]. However, the question of how to quantify ENSO predictability remains the subject of debate[5,10]. Many scientists adopted linear singular vector (LSV) to quantify ENSO predictability[3], while it holds only under the condition that the initial perturbations are sufficiently small and the prediction time intervals are very short. Therefore, LSV cannot describe the effect of nonlinearity on ENSO predictability[11—13].

Three types of predictability problems proposed by Mu et al.[5] are established on the basis of nonlinear model itself without any linear approximation, which can therefore reveal the effect of nonlinearity of predictability. In this paper, we investigate the applications of these three types of predictability problems to quantifying ENSO predictability. With a theoretical model, we derive the nonlinear optimization problems of estimating the lower bounds of the maximum

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predictable time and the maximum allowable initial errors for ENSO events, and the upper bound of the maximum prediction error. By solving these problems, it is expected that we can reveal the law of error evolution for ENSO events, and explore the difference between the linear and nonlinear estimations of the prediction error.

1 Model equations

With a number of simplifications, Wang and Fang[6] (hereafter referred to as WF96) distilled an intermediate coupled ocean-atmosphere model[7] to a theoretical model. This theoretical model consists of only two time-dependent equations, i.e. sea surface temperature anomaly (SSTA) T and thermocline depth anomaly h equations.

$$\begin{cases} \dfrac{\mathrm{d}T}{\mathrm{d}t} = a_1 T - a_2 h + \sqrt{\dfrac{2}{3}}\,T(T - a_3 h), \\ \dfrac{\mathrm{d}h}{\mathrm{d}t} = b(2h - T), \end{cases} \tag{3.1}$$

where $a_1 = (\overline{T}_z + \overline{T}_x - \bar{u}_1 - \alpha_s)$, $a_2 = (\mu + \delta_1)\overline{T}_x|$, $a_3 = \mu + \delta_1$, and $b = \dfrac{2\alpha}{p(1-3\alpha^2)}$. Here, the coefficients a_1 and a_2 involve the basic state parameters $\overline{T}_x$ and $\overline{T}_z$, which characterize respectively the mean temperature difference between the eastern and western Pacific and between the surface and subsurface water. Note that these basic state parameters vary with time, reflecting the annual cycle of the basic state. Two dimensionless coupling parameters are presented in this model. One is the air-sea coupling coefficient, $\alpha = \left(\dfrac{L_0}{L_y}\right)^2$, where L_0 is the oceanic Rossby radius of deformation and L_y is the characteristic meridional length scale of the coupled ENSO mode. For $L_0 = 300$ km, when L_y varies from 1000 km to 400 km, α increases from 0.09 to 0.5625. Another coupling parameter is $\mu = \dfrac{\mu H_1}{\theta}$, which measures the degree of coupling between thermocline fluctuation and SST. The meanings and typical values of the other parameters are listed in Table 1 of Ref. [6].

Wang and Fang[6] demonstrated that the WF96 model described the interannual variation of SSTA and the thermocline depth anomaly in the Nino-3 region, which highlights the cyclic, chaotic, and season-dependent evolution of ENSO. Duan et al.[14] used this model to study the initial pattern that evolves into ENSO most potentially, i.e. the optimal precursors for ENSO. The theoretical results discover a general law of oceanic motions: the negative (positive) SSTA and positive (negative) thermocline depth anomalies averaged over the whole Nino-3 region always persist for about 3—4 months before the onset of El Nino (La Nina). That is to say, the negative (positive) SSTA and positive (negative) thermocline depth anomalies averaged over equatorial east Pacific act as the optimal precursor for ENSO qualitatively.

In this paper, we will use this model to investigate the applications of nonlinear optimization methods to quantifying the predictability of ENSO event. It should be pointed out that the ENSO event considered in this paper is a model ENSO event, which is based on a particular model. Model ENSO event is obtained by running the model with an initial anomaly. In the rest of the paper, to facilitate the discussion, we simply call it ENSO event and does not emphasize the "model" ENSO. The ENSO model is solved numerically by using a fourth-order Runge-Kutta scheme, where the time step $\mathrm{d}t = 0.01$ represents one day.

2 The lower bound of the maximum predictable time for ENSO event

In this paper, WF96 model is assumed to be perfect, and the predictability limitation is therefore considered to be set only by the uncertainty of the initial condition.

Let M_t be the propagator of WF96 model, which takes perturbations at the initial time and "propagates" these perturbations to some time in the future. U_0 is the initial observation of an ENSO event, and $u_0 = (T', h')$ is an initial perturbation of U_0. The norm $\| u(t) \| = \max\{|T'(t)|, |h'(t)|\}$ is chosen to measure the growth of the initial perturbation. For the constraint condition $\| u_0 \| \leqslant \sigma$, we can derive the lower bound of the maximum predictable time for the model ENSO event, where σ is the bound of the initial observational error,

$$\zeta_{\varepsilon^0} = \min_{\| u_0 \| \leqslant \sigma} \{\zeta_{u_0} \mid \zeta_{u_0} = \max\tau, \ \| M_t(U_0 + u_0) - M_t(U_0) \| \leqslant \varepsilon^0, \ 0 \leqslant t \leqslant \tau\}. \tag{1}$$

We choose dimensionless $(-0.1, 0.1)$ and $(0.1, -0.1)$ of (T_0, h_0) as the initial values of the basic states, which respectively evolve into an El Nino and a La Nina events (Fig. 1). Here, (−0.1, 0.1) corresponds to the dimensional SSTA ($\hat{T}$) −0.21℃ and thermocline depth anomaly ($\hat{h}$) 5 m, and (0.1, −0.1) corresponds to (0.21℃, −5 m) of ($\hat{T}$, $\hat{h}$).

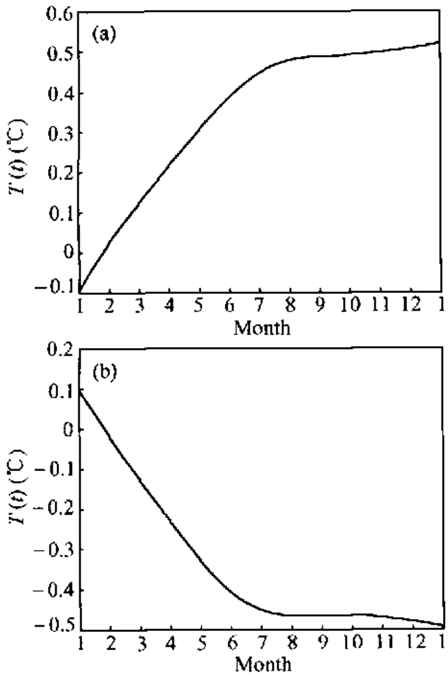

Fig. 1. (a) El Nino with initial observation (−0.1, 0.1) and (b) La Nina with initial observation (0.1, −0.1).

For a predetermined initial observational error bound σ, and the maximum allowable prediction error ε^0, the lower bound of the maximum predictable time of the above El Nino and La Nina events can be computed from Eq. (1). Firstly, we investigate the case of the initial time being January. Tables 1 and 2 show the lower bound of the maximum predictable time for the El Nino and the La Nina events, respectively, where ζ_{ε^0} represents the lower bound of the maximum predictable time.

Table 1. Lower bound of the maximum predictable time for El Nino with the initial time being January

ε^0	ζ_{ε^0}				
	$\sigma=0.06$	$\sigma=0.07$	$\sigma=0.08$	$\sigma=0.09$	$\sigma=0.10$
0.35	267	204	173	154	139
0.40	317	216	183	161	146
0.45	403	229	193	169	152
0.50	420	246	204	177	159

To further investigate the predictability of El Nino and La Nina, Fig. 2 describes the difference of ζ_{ε^0} between El Nino and La Nina. It is shown that for the sufficiently small initial observational error

Table 2. Lower bound of the maximum predictable time for La Nina with the initial time being January

ε^0	ζ_{ε^0}				
	$\sigma=0.06$	$\sigma=0.07$	$\sigma=0.08$	$\sigma=0.09$	$\sigma=0.10$
0.35	1031	1019	765	639	524
0.40	1036	1023	781	648	531
0.45	1040	1026	957	659	538
0.50	1045	1029	1021	672	546

bound, with the maximum allowable prediction error increasing from 0.35 to 0.50, El Nino event can be forecasted bestriding spring for a long time. For example, if the initial observational error bound $\sigma=0.05$, which means that the initial observational error bound of SSTA is less than 0.1℃, and the maximum allowable prediction error $\varepsilon^0=0.50$ (i.e. the maximum allowable prediction error is not larger than 1.0℃), the maximum predictable time for El Nino is at least 425 days. That is to say, WF96 model can predict El Nino bestriding spring for 245 days. For the large initial observational error bound, the lower bound of the maximum predictable time for El Nino is always limited in the range from about 90 days to 180 days, which corresponds to the season of April-June. WF96 model cannot forecast El Nino bestriding spring with the maximum allowable prediction error. For instance, when the initial observational error bound $\sigma=0.09$, the dimensional initial observational error bound of SSTA is not larger than 0.18℃, and the lower bound of the maximum predictable time for El Nino is 172 days. For La Nina event with given allowable maximum prediction error, it can be forecasted bestriding spring for a long time by WF96 model and the spring predictability barrier does not occur. From the above numerical results, it can be found that for the given initial observational error bound and the allowable maximum prediction error, WF96 model can predict La Nina bestriding spring for at least one year. For example, in the case of $\sigma=0.1$ and $\varepsilon^0=0.50$, the maximum predictable time for La Nina is at least 546 days.

Besides, the case of the initial time being October is also investigated (Fig. 2(b)). It can be shown that when the initial observational error bound is sufficiently small, $\sigma\in[0.01, 0.05]$ (compared to the above initial observational error bound $\sigma\in[0.06, 0.10]$), which means that the dimensional initial observational error bound of SSTA is constrained to the range of [0.02℃, 0.10℃], and the WF96 model cannot forecast El Nino bestriding next spring. That is to say, with uncertain initial fields, El Nino event

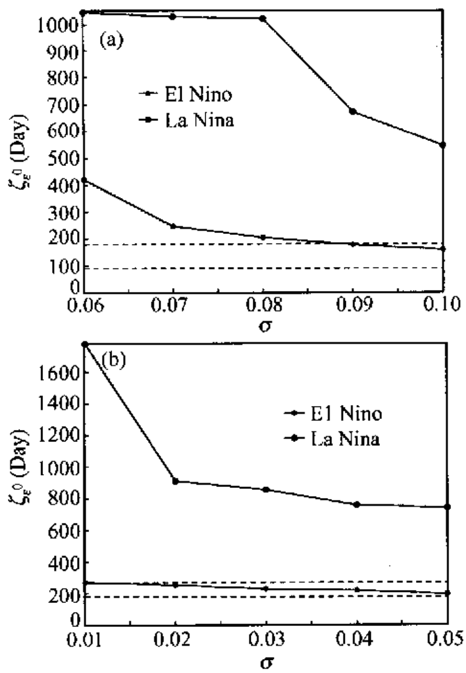

Fig. 2. The lower bound of the maximum predictable time for ENSO. (a) The initial time is January; (b) the initial time is October. The time periods marked by two dash-lines are spring, and the allowable prediction error is 0.50.

cannot be forecasted bestriding next spring from last October by WF96 model for the allowable prediction precision. Whereas for La Nina event, there does not occur the spring predictability barrier and it can be forecasted through next spring from last October. The above analyses indicate that El Nino is less predictable than La Nina.

3 The upper bound of the maximum prediction error for ENSO event

In this section, the predictability of WF96 model bestriding spring is investigated by studying the prediction error of model ENSO events.

Suppose that the initial observational error bound σ and the prediction time τ are known, the upper bound of the maximum prediction error for ENSO events can be derived by the following nonlinear optimization problem

$$E_{\mathrm{u}} = \max_{\| u_0 \| \leqslant \sigma} \| M_{\tau}(U_0 + u_0) - M_{\tau}(U_0) \| , \quad (2)$$

where U_0 is the initial observation of an ENSO event, and u_0 is an initial perturbation. Mu et al.[12] and Mu and Duan[13] treated U_0 in Eq. (2) as the initial values of realistic ENSO events, and then regarded the initial perturbation $u_{0\delta}$ satisfying Eq. (2) as the conditional nonlinear optimal perturbation (CNOP) of ENSO, which is characterized by the maximum nonlinear growth of the initial perturbations satisfying $\| u_0 \| \leqslant \sigma$. Consequently, CNOP has the largest effect on the prediction results of ENSO events at prediction time τ. Details can be found in Refs. [12—14].

Predetermining the maximum allowable prediction error (prediction precision) $\varepsilon^0 = 0.50$ (i.e. the dimensional maximum allowable prediction error of SSTA is not larger than 1.0℃), we consider the ENSO forecast initialized in January and ended in October, November, and December, respectively. For the initial observational error bound $\sigma \in [0.06, 0.10]$, which corresponds to the dimensional SSTA observational error bound [0.12℃, 0.21℃], we estimate the upper bound of the maximum prediction error for ENSO events and evaluate the forecast ability of WF96 model bestriding spring.

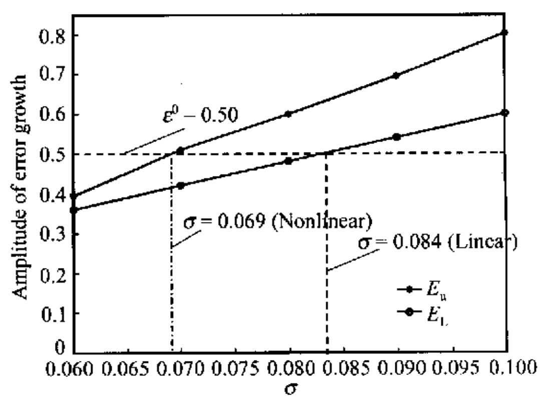

Fig. 3. The upper bound of the maximum prediction error for El Nino E_{u}, and its linear estimation E_{L}. The time interval is from January to October.

Figure 3 shows the prediction error of El Nino as a function of σ with the time interval from January to October, where E_{L} is the linear evolution of the above mentioned initial perturbation $u_{0\delta}$ satisfying Eq. (2), which is obtained by integrating the tangent linear model (TLM) with the initial value $u_{0\delta}$. It can be shown that for the given maximum allowable prediction error $\varepsilon^0 = 0.50$, when $\sigma \leqslant 0.07$ (the dimensional SSTA observational error bound is less than 0.14℃), WF96 model can potentially forecast El Nino bestriding spring, while when the SSTA initial observational error bound exceeds 0.14℃, El Nino cannot be forecasted bestriding spring by WF96 model. However, information on the linear estimation of the prediction error by TLM suggests that when the initial SSTA observational error bound is less than 0.17 ℃, the model prediction error still satisfies the given prediction precision $\varepsilon^0 = 0.50$. That is to say, even though the initial observational error bound exceeds 0.14 ℃, the model can also forecast El Nino bestriding spring. Obviously, it is not true. Since the nonlinear model is assumed to describe the observed ENSO events and the TLM is only an approximation to it, the estimation of the prediction error by TLM approximates merely the evolution of the prediction error for ENSO, which cannot quantitatively reflect the nonlinear evolution of the prediction error for WF96 model. Thus, estimating the prediction error by TLM may cause the false evaluation of the model' s forecast ability. For the time interval from January to November and December, there are quite similar results for the above El Nino event. For simplicity, we do not discuss it in detail.

Additionally, we also perform extensive numerical experiments to investigate the prediction error of El Nino taking October as the initial time. It is demonstrated that for sufficiently small initial observational error bound, the upper bound of the maximum prediction error of El Nino bestriding spring is always larger than the above allowable prediction precision. Therefore, for El Nino forecast initialized in October, prediction uncertainties are more likely to occur when it bestrides next spring.

The above discussion demonstrates that the predictability of WF96 model about El Nino depends on the initial time of forecast. If the forecast is initialized in January, the WF96 model may be of certain predictability of El Nino bestriding spring. But if the forecast initialized in October, the forecast skill of WF96 model about El Nino may dramatically decline. Additionally, the above results also explore the difference between linear and nonlinear estimation of prediction error and reveal the limitation of linear approach.

For La Nina event, we also conduct many numerical experiments. The results suggest that for the above initial observational error bound ($\sigma \in [0.06, 0.10]$), WF96 model is always of strong forecast skill whatever the initial time is. That is to say, for the given prediction precision, WF96 model can always forecast La Nina bestriding spring, which further demonstrates that El Nino is less predictable than La Nina.

4 The lower bound of the maximum allowable initial error of ENSO events

The above analysis has shown that for the known initial observational error bound and the prediction precision, we can evaluate the forecast ability of WF96 model about ENSO events bestriding spring by estimating the maximum predictable time for ENSO; while for the given initial observational error bound and prediction time, we can also estimate the predictability of model ENSO events bestriding spring by computing the upper bound of the maximum prediction error of ENSO. Contrarily, if it is expected that the model can forecast ENSO bestriding spring, what conditions should the initial observation of ENSO satisfy, or say, how small should the initial observation error be?

To discuss this problem, we establish the lower bound of the maximum allowable initial observational error for ENSO events,

$$\bar{\sigma}_\tau = \max_\sigma \{\sigma \mid \| M_\tau(U_0 + u_0) - M_\tau(U_0) \| \leqslant \varepsilon^0, \ \| u_0 \| \leqslant \sigma\}, \quad (3)$$

where ε^0 is the maximum allowable prediction error, U_0 is the initial observation of ENSO, and u_0 is an initial perturbation of ENSO.

Table 3 describes the lower bounds of the maximum allowable initial observational errors for El Nino, where the time interval τ is taken as from January to August, October, and December. It is demonstrated that for the prediction precision $\varepsilon^0 = 0.50$ (the maximum allowable prediction error of SSTA is not larger than 1.0 ℃), when WF96 model forecasts El Nino during August, the lower bound of the maximum allowable initial observational error bound is $\sigma = 0.0704$, which means that the initial SSTA observational error bound cannot exceed 0.14 ℃. That is to say, if we want to forecast El Nino during August using the WF96 model, the initial observational error bound of SSTA should be less than 0.14 ℃. Otherwise, due to the fast growth of initial perturbation during spring, the prediction error will exceed the given prediction precision, and the phenomenon of spring predictability barrier will occur.

For El Nino forecast starting with October, we also calculate the lower bound of the maximum allowable initial observational error for El Nino. The results demonstrate that for the given prediction precisions, the lower bounds of the maximum allowable initial observational errors for El Nino are always less than 0.01, corresponding to the dimensional initial observational error bound 0.02℃, which cannot be generally identified in observation. That is to say, the observation cannot usually reach the precision of 0.02℃[15]. Therefore, WF96 model cannot forecast El Nino bestriding next spring from October for the given prediction precision.

Table 3. The lower bound of the maximum allowable initial observational error for El Nino event

τ	$\bar{\sigma}_\tau$			
	$\varepsilon^0=0.35$	$\varepsilon^0=0.40$	$\varepsilon^0=0.45$	$\varepsilon^0=0.50$
Jan-Aug	0.0633	0.0654	0.0686	0.0704
Jan-Oct	0.0586	0.0606	0.0644	0.0673
Jan-Dec	0.0516	0.0551	0.0614	0.0641

For La Nina event, discussions in the above two sections have demonstrated that the phenomenon of spring predictability barrier does not occur, and with the given prediction precision, the lower bounds of the maximum allowable initial observational errors for La Nina should be larger than those for El Nino. Actually, our numerical results verify this theoretical conclusion. This also sheds lights on that La Nina is more predictable than El Nino from another point of view.

5 Summary and discussion

In this paper, we have established the nonlinear optimization problems related to the lower bound of the maximum predictable time for ENSO, the upper bound of the maximum prediction error, and the lower bound of the maximum allowable initial error. By solving these problems, the phenomenon of "spring predictability barrier" for ENSO has been revealed, which shows that ENSO predictability depends on the initial time when the forecast started. Further studies demonstrate that the large error of the initial field potentially induces the prominent spring predictability barrier. There is also evidence that La Nina is more predictable than El Nino, which qualitatively supports the view of Moore and Kleeman[16] about ENSO predictability. Then in the present study, the effects of nonlinearity on predictability are further emphasized and the limitation of the tangent linear model (TLM) estimating prediction error is pointed out. TLM is only an approximation to the nonlinear model, which holds only when the initial perturbations are sufficiently small and the optimization time intervals are very short. For the finite amplitude of initial perturbations and the long optimization time intervals, if we adopt TLM to obtain the information on the prediction error, it may result in a false judgement on the prediction results of nonlinear model. This is not favorable for the improvement of the ENSO forecast skill. Therefore, when the prediction error is used to evaluate the model predictability, we would rather believe the information on prediction error obtained by the nonlinear model.

These above results are derived theoretically from a numerical model for ENSO by identifying this model the so-called "perfect model scenario". Given a perfect model, the model ENSO may represent the realistic ENSO event. In this case, ENSO predictability is limited only by the growth of the uncertainties in the initial condition. The model ENSO predictability and the predictability of realistic ENSO are unified. Upon this assumption of "perfect model", the evolutions of the initial uncertainties of ENSO are quantified through nonlinear optimization method in the present study and some indicative results are obtained. Practical predictability experiments are often carried out with an imperfect model forecasting observational data. Model error exists in the particular model employed. The predictability quantified by numerical models may under- or overestimate the inherent ENSO predictability. That is to say, the different numerical models may show different predictabilities for ENSO due to the model error.

In this paper, the ENSO model employed is simply and may be of some uncertainties in describing ENSO oscillation. Therefore, the derived results from this model are qualitatively indicative. The numerical experiments performed here are of exploratory nature. However, we are greatly encouraged by these results. It is expected that for a more realistic model, there will be more significant findings by using the nonlinear optimization methods. The methods may verify the results derived by a simple model and quantify the ENSO predictability.

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