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Applications of Conditional Nonlinear Optimal Perturbation in Predictability Study and Sensitivity Analysis of Weather and Climate

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ABSTRACT

Considering the limitation of the linear theory of singular vector (SV), the authors and their collaborators proposed conditional nonlinear optimal perturbation (CNOP) and then applied it in the predictability study and the sensitivity analysis of weather and climate system. To celebrate the 20th anniversary of Chinese National Committee for World Climate Research Programme (WCRP), this paper is devoted to reviewing the main results of these studies. First, CNOP represents the initial perturbation that has largest nonlinear evolution at prediction time, which is different from linear singular vector (LSV) for the large magnitude of initial perturbation or/and the long optimization time interval. Second, CNOP, rather than linear singular vector (LSV), represents the initial anomaly that evolves into ENSO events most probably. It is also the CNOP that induces the most prominent seasonal variation of error growth for ENSO predictability; furthermore, CNOP was applied to investigate the decadal variability of ENSO asymmetry. It is demonstrated that the changing nonlinearity causes the change of ENSO asymmetry. Third, in the studies of the sensitivity and stability of ocean's thermohaline circulation (THC), the nonlinear asymmetric response of THC to finite amplitude of initial perturbations was revealed by CNOP. Through this approach the passive mechanism of decadal variation of THC was demonstrated; Also the authors studies the instability and sensitivity analysis of grassland ecosystem by using CNOP and show the mechanism of the transitions between the grassland and desert states. Finally, a detailed discussion on the results obtained by CNOP suggests the applicability of CNOP in predictability studies and sensitivity analysis.

Key words: predictability, weather, climate, optimal perturbation

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1. Introduction

Predictability and sensitivity studies of atmospheric or oceanic motions are of great significance in the theoretical studies and the operational implementation of numerical weather and climate prediction (Lorenz, 1965, 1969, 1975), whose results can be used to guide the developing of the numerical forecast model, then improving the forecast skill. In these fields, one approach based on linear fastest growing perturbation has been widely used to attack these problems, i.e., linear singular vector (LSV) (Lorenz, 1965). For example, LSV was used to determine the initial condition for optimal growth in a coupled ocean-atmosphere model of El Niño-Southern Oscillation (ENSO), in an attempt to explore error growth and predictability of the coupled model (Xue et al., 1997a, b; Thompson, 1998; Samelson and Tziperman, 2001; Moore and Kleeman, 1996). Tziperman and Ioannou (2002) also used this approach to find a possible physical mechanism of transient amplification of initial perturbations to the thermohaline circulation (THC) of the oceans, etc. Furthermore, this method can be applied to the blocking problem (Buizza and Molteni, 1996; Frederisen, 1997; Luo et al., 2001). Obviously, LSV has become a widely used tool in the studies of weather and climate predictability.

Note that LSV is the fastest growing perturbation of tangent linear model. Due to the absence of nonlinearity, it cannot guarantee the role of initial uncertainties having largest effect on prediction results (Mu, 2000; Mu and Wang, 2001; Mu et al., 2003; Mu

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and Zhang, 2006). The motions of atmospheric and oceanic flows are generally nonlinear. Consequently, the derived results by LSV remains questionable in nonlinear model (Mu et al., 2003). The nonlinearity limits the applications of LSV to the predictability studies and sensitivity analysis of weather and climate system.

Considering the limitation of LSV, Mu et al. (2003) proposed a novel concept of conditional nonlinear optimal perturbation (CNOP) by using directly the nonlinear model, which represents the initial perturbation that has the largest nonlinear evolution at the prediction time for the initial perturbations satisfying certain physical constraint condition. CNOP is different from LSV for the finite amplitude of initial perturbations and/or the long time intervals (Mu et al., 2003; Mu and Duan, 2003; Mu et al., 2004; Duan et al., 2004; Sun et al., 2005; Mu and Zhang, 2006). The property of the fastest growth of LSV lies in the measurement of the linear growth rate of the initial perturbations, while CNOP is measured by the nonlinear evolution at prediction time. The essence of its maximum nonlinear evolution make it represent the initial uncertainties that has largest impact on forecast results and play a more important role than LSV in the predictability studies and sensitivity analysis (Mu and Duan, 2005; Duan and Mu, 2005).

With these above in mind, the authors and their collaborators used CNOP to investigate the predictability and sensitivity of weather and climate. CNOP has been used to determine the optimal initial perturbation of coupled ENSO model, in attempt to find the optimal precursor of ENSO and the initial error that have largest effect on ENSO prediction (Duan et al., 2004; Mu and Duan, 2003; Mu et al., 2006). Mu et al. (2004) also applied this approach to investigate the sensitivity and stability ocean's thermohaline circulation (THC).

All these above works on CNOP demonstrate the applicability of CNOP. The concept and the properties of CNOP will be introduced briefly in next section. In section 3, we will review the application of CNOP to the studies of ENSO predictability. And in section 3, the studies of the sensitivity and stability of THC are summarized in detail. Also the instability and sensitivity of grassland system are presented in section 4, and a detailed discussion is given in final section.

2. Conditional nonlinear optimal perturbation and its properties

Consider a set of nonlinear partial differential equations describing the evolution of a state vector $w(t)$:

$$\begin{cases} \dfrac{\partial \boldsymbol{w}}{\partial t} + F(\boldsymbol{w}) = 0, \\ \boldsymbol{w}|_{t=0} = \boldsymbol{w}_0, \end{cases} \quad \text{in} \quad \Omega \times [0, T] \tag{1}$$

where t is the time, $\boldsymbol{w}(\boldsymbol{x},t) = (w_1(\boldsymbol{x},t), w_2(\boldsymbol{x},t), \ldots, w_n(\boldsymbol{x},t))$ is the state vector and F is a nonlinear differentiable operator. Furthermore, $\boldsymbol{w}_0$ is the initial state, $(\boldsymbol{x},t) \in \Omega \times [0,T]$, Ω is a domain in R^n, $T < +\infty$, and $\boldsymbol{x} = (x_1, x_2, ..., x_n)$. Assuming that the evolution equations and the initial state are known exactly, the future state can be determined by integrating Eq. (1) with the appropriate initial condition. The solution to Eq. (1) for the state vector w at time τ is given by

$$\boldsymbol{w}(\boldsymbol{x},\tau) = M_\tau(\boldsymbol{w}_0). \tag{2}$$

Here, M_τ is the propagator, which, as described by (2), "propagates" the initial value to the time τ in the future. Let $\boldsymbol{U}(\boldsymbol{x},t)$ and $\boldsymbol{U}(\boldsymbol{x},t) + \boldsymbol{u}(\boldsymbol{x},t)$ be the solutions of problem (1) with initial value $\boldsymbol{U}_0$ and $\boldsymbol{U}_0 + \boldsymbol{u}_0$ respectively, where $\boldsymbol{u}_0$ is the initial perturbation. We have

$$\boldsymbol{U}(\tau) = M_\tau(\boldsymbol{U}_0), \quad \boldsymbol{U}(\tau) + \boldsymbol{u}(\tau) = M_\tau(\boldsymbol{U}_0 + \boldsymbol{u}_0). \tag{3}$$

So $\boldsymbol{u}(\tau)$ describes the evolution of the initial perturbation $\boldsymbol{u}_0$.

For a chosen norm $\|\cdot\|$, an initial perturbation $u_{0\delta}$ is called CNOP, if and only if

$$J(u_{0\delta}) = \max_{\|u_0\| \le \delta} J(u_0),$$

where

$$J(u_0) = \|M_\tau(U_0 + u_0) - M_\tau(U_0)\|, \tag{4}$$

$\|u_0\| \le \delta$ is constraint condition of initial perturbations with the chosen norm. Obviously, we can also investigate the situation that the initial perturbations belong to a kind of functional set. Furthermore, the constraint condition could be some physical laws that initial perturbation should satisfy, etc.

From Eq. (4), it is easily derived that CNOP is the initial perturbation whose nonlinear evolution attains the maximal value of the functional J at time τ (Mu et al., 2003; Mu and Duan, 2003). Mathematically, it is the global maximum of the objective function in the phase space. In some cases, there exists local maximum of the objective function. And the corresponding initial perturbations are referred to local CNOP. CNOP and local CNOP are both significant in the studies of predictability and sensitivity analysis, which will be discussed in detail in the subsequent several sections.

CNOP is related to a nonlinear optimization problem, so it is difficult to solve it analytically. Mu et al. (2003), Duan et al. (2004), Mu et al. (2004), Sun et al. (2005), etc. used the simple model consisting of a set of ordinary differential equations (ODEs) and solved numerically the CNOP by an optimization alg-

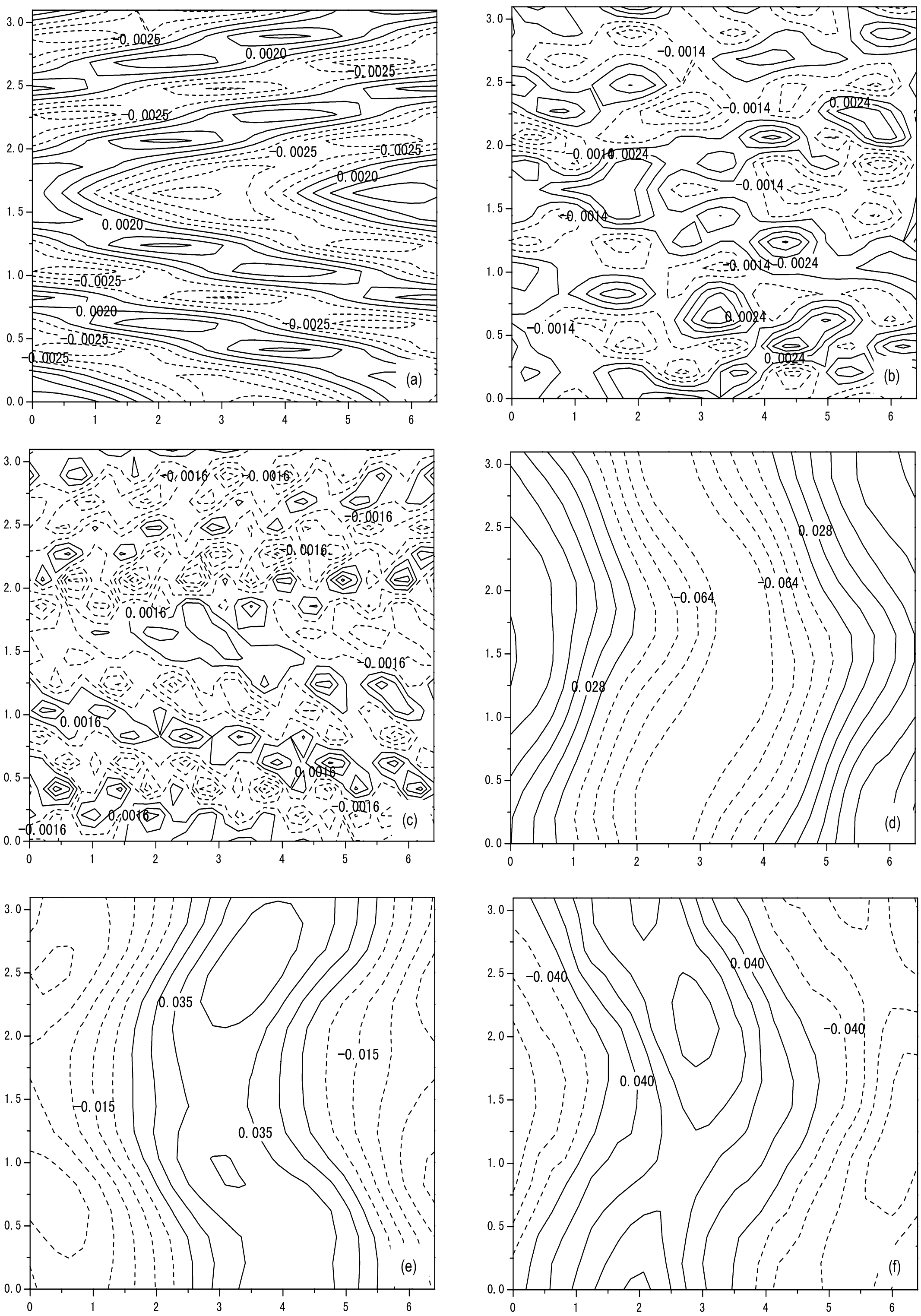

Fig. 1. LSV, local and global CNOPs for 5 days. The interval values of contour in the (a), (b), (c), (d), (e) and (f) are 0.0015, 0.0019, 0.0016, 0.023, 0.0125, and 0.04, respectively. (from Mu and Zhang, 2006).

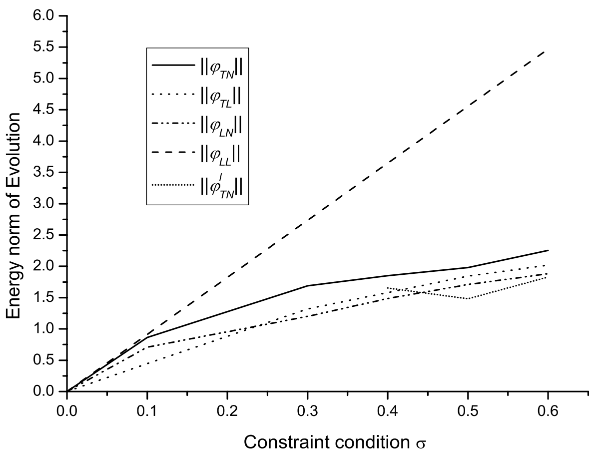

Fig. 2. Linear and nonlinear evolutions of CNOP and LSV for 5 days. $\|\varphi_{\rm TN}\|$ and $\|\varphi_{\rm TL}\|$ are the norms of nonlinear and linear evolutions of CNOP, respectively; $\|\varphi_{\rm LN}\|$ and $\|\varphi_{\rm LL}\|$ are the norms of nonlinear and linear evolutions of LSV, respectively. $\|\varphi_{l,{\rm TN}}\|$ is the norm of nonlinear evolution of local CNOP. The figure is from Mu and Zhang (2006).

orithm of the sequential quadratic programming (SQP) method (Powell et al., 1982). Some common characteristics of CNOP were found. First, when linear approximation is not valid, CNOP is significantly different from LSV. Second, in some cases, there exists local CNOPs with clear physical meaning. Third, the CNOPs and local CNOPs are all located on the boundary of the domain defined by the given constraint condition in phase space. These characteristics may form the intrinsic properties of CNOP. Considering that the governing equations of the atmospheric and oceanic motions are generally partial differential equations (PDEs), Mu and Zhang (2006) employed a two-dimensional quasigeostrophic model to compute the CNOPs of the basic states with different stability properties, in attempt to reveal the above characteristics of CNOP. It is shown that when the initial perturbations are large, or the time periods are considerably long, or both, CNOPs and LSVs show remarkable difference in two aspects. First, although both CNOP and LSV are initial perturbation fields, LSV stands for the optimal growing direction, while CNOP represents the initial perturbation which will have largest effects at the end of the time period under proper physical constraints. CNOP is only a "pattern" rather than a "direction" due to the nonlinearity of the model. Figure 1 shows the stream function of LSV, local CNOP and their corresponding nonlinear evolutions, respectively. It is clear that when TLM is not a good approximation, CNOPs (local CNOP) and the corresponding LSVs, which have the same magnitudes as CNOPs, are remarkably different. These CNOPs and local CNOPs are located on the boundary of the constraint condition. Second, the difference between CNOPs and LSVs is also shown in their linear and nonlinear evolutions (Fig. 2). It is demonstrated that when the initial perturbations are sufficiently small, the evolutions of LSV and CNOP are trivially different. With the increasing magnitude of initial perturbations, the differences between the linear and nonlinear evolutions of CNOP (LSV) and between the nonlinear evolution of CNOP and linear evolution of LSV become more and more considerable. Thus, the three common characteristics of CNOP derived by simple models of ODEs are also occurred in the models consisting of the PDEs. This indicates that these three characteristics are inherent for CNOP. Simultaneously, the results of Mu and Zhang (2006) imply the feasibility of CNOP in realistic numerical models.

3. Applications of CNOP in the studies of ENSO predictability dynamics

The studies have shown that it is of great significance for improving ENSO predictability to find out the precursors for ENSO events and to explore the mechanism of the initial error growth (Moore and Kleeman, 1996; Thompson, 1998, etc.). Although LSV has been used to explore these problems, LSV is limited in revealing the effect of nonlinearity. Considering this point, the authors adopted CNOP to investigate these problems.

3.1 *Optimal precursors of ENSO events*

Duan et al. (2004) employed CNOP to study the optimal precursors for ENSO events by in the couple model of Wang and Fang (1996) (WF96). In the study, a simple coupled ocean-atmosphere model is adopted. It is shown that for the climatological basic annual cycle, regardless of what the initial time is, there exist CNOP and local CNOP. These CNOPs (local CNOPs) are all located on the boundary of the constraint condition and have robust patterns of negative (positive) sea surface temperature anomaly (SSTA) and positive (negative) thermocline depth anomaly qualitatively. When the initial perturbations are large, CNOP and local CNOP are respectively quite different from the LSVs under the condition that they are of the same amplitude of norm. And with the amplitude of the initial perturbations increasing, the differences between the CNOPs (local CNOPs) and LSVs become increasingly considerable.

Duan et al. (2004) also investigated the evolution of CNOP (local CNOP) and LSV of annual cycle. It is demonstrated that for the short optimization time interval, no matter what the initial time is, there are only trivial differences between the linear and nonlinear evolution of the SSTA component of the CNOP (local CNOP), and between those of LSVs respectively. For the long time interval and large amplitude of initial perturbations, they have considerable differences respectively. Besides, the differences of the nonlinear evolution of LSVs and CNOPs (local CNOP) were also explored. For the same and large amplitude initial perturbations, the nonlinear evolution of CNOP is significantly larger than that of LSV, which indicates that CNOP is optimal compared to LSV under the condition that they are of the same amplitude of norm. Further analysis demonstrates that the CNOP of annual cycle evolves into the positive SSTA nonlinearly, which takes a striking resemblance to the development of El Niño. In fact, it acts as a precursor for El Niño event in WF96 model. Although the corresponding LSV also develops into an El Niño, the intensity is considerably weaker than that of CNOP. In this sense, they regarded CNOP as the optimal precursor for El Niño. For the local CNOP of annual cycle, its nonlinear evolution is only a little larger than that of the corresponding scaled LSV. This phenomenon can be explained by the locality of the local CNOP. As to the physical characteristic local CNOP bears, after investigating the nonlinear evolution, it is found that local CNOP acts as the optimal precursor of La Niña event in this simple ENSO model.

In the end of Duan et al. (2004), the authors compared the intensity of El Niño with that of La Niña. They found that when using LSV to study the intensity of ENSO events, the corresponding El Niño and La Niña events in TLM are of the equal amplitude, or say, the El Niño and La Niña events are symmetric about climatological mean state. While in CNOP approach, the El Niño event is obviously stronger than the La Niña event under the condition that the initial CNOP and local CNOP are of the same amplitude, which is quite consistent with the observation. Clearly, the linear theory of singular vector cannot reveal the nonlinear asymmetry of El Niño and La Niña. The reason is that the El Niño-La Niña asymmetry is caused by a nonlinear feedback of the model. The above theoretical results are quite consistent with the 22-year NCEP reanalysis data qualitatively .

In view of the simplicity of the above theoretical model, an intermediate coupled ENSO model of Zebiak and Cane (1987) (CZ model) is also used to investigate the precursors for ENSO (Xu, 2006). The extensive numerical experiments demonstrate that the CNOPs, rather than LSVs, of the climatological basic-state annual cycle acts as the optimal precursors of ENSO (Figs. 3 and 4), whose configuration of SSTA and thermocline depth anomaly reveals the fact that the transition phase of thermocline depth displacement leads to the SST variation and supports the results of Duan et al. (2004). Then the results obtained by CZ model further show the spatial structure of the CNOP and emphasize the locality of optimal precursors for ENSO in spatial distribution. Besides, the effect of nonlinearity occurred in the model on the ENSO evolution is also explored. The results suggest that the nonlinear temperature advection plays a dominant role in ENSO evolution. Practically, the nonlinear temperature advection enhances El Niño and suppresses La Niña, then resulting in the asymmetry, which verifies the results derived by the theoretical model and then indicates that ENSO asymmetry occurred in observation may results mainly from the effect of nonlinear temperature advection.

3.2 *Seasonal dependence of error growth related to ENSO predictability*

Considering that many state-of-the-art coupled ocean-atmosphere model have particular difficulty in the prediction of ENSO prior to boreal spring and the causes of this spring predictability barrier (SPB) remain controversial and elusive, the authors used the above mentioned simple model to investigate this problem from the point of view of initial error growth using CNOP approach. The results show that the largest growth rate of the CNOP for El Niño occurs during boreal spring, which is referred to the April-May-June (AMJ) period and coincides with the time of the predictability barrier of ENSO models. With increasing magnitudes of CNOPs, the amplitude of spring error growth for El Niño becomes progressively large. Although the largest error growth of El Niño during AMJ is also shown by LSV, the CNOP growth is significantly larger than the corresponding LSV growth for large-amplitude initial perturbations in some model El Niño events. This has the implication that the nonlinearity plays an important role in error growth of ENSO warm event. Furthermore, we compared the seasonal variations of the CNOP growth for ENSO events with those of a large ensemble of initial errors chosen randomly from a constrained initial domain. It is demonstrated that not all initial errors tend to induce prominent seasonal variation of error growth, it is the CNOP of El Niño that exhibits the most prominent seasonal variation. But for the La Niña events, even if the initial errors are taken to be of the types of CNOPs, their evolutions do not tend to exhibit the prominent seasonal dependence. These imply that the seasonal variation of error growth for

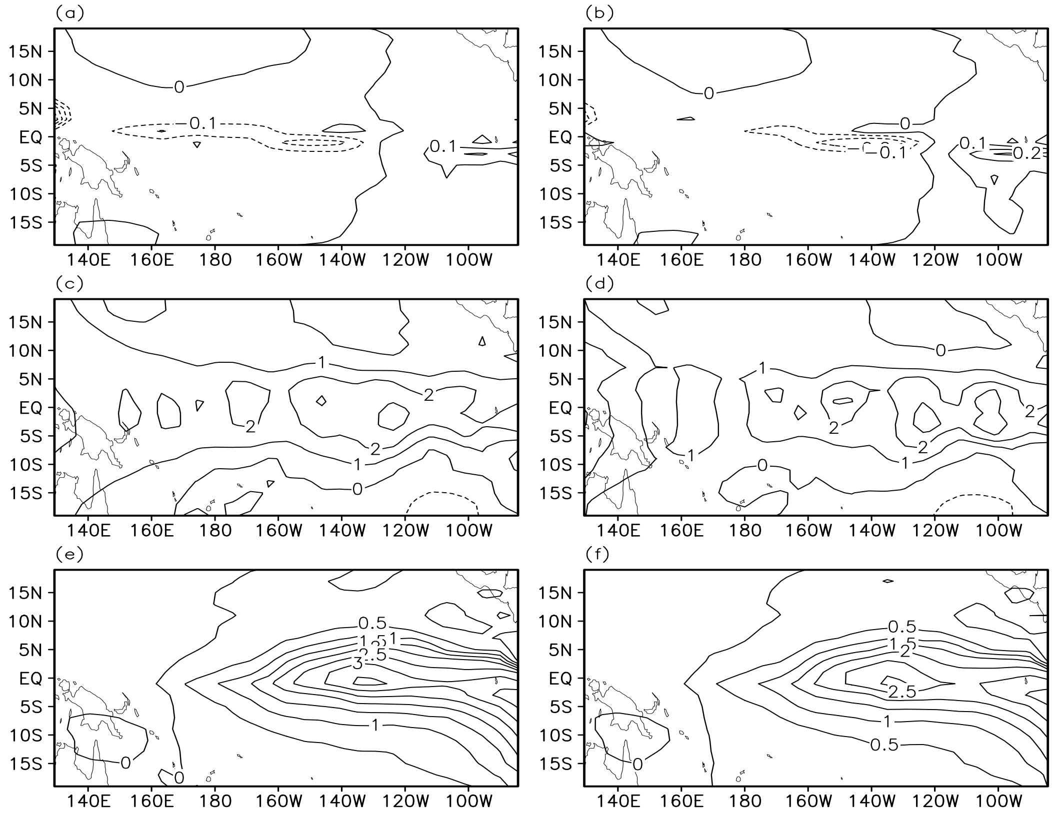

Fig. 3. Comparison between CNOP and LSV. The left (right) column corresponds to CNOP (LSV). (a), (c) and (e) are respectively the SSTA, thermocline depth anomaly, and the SSTA component of the nonlinear evolution for CNOP. (b), (d) and (f) are those of LSV. The El Niño induced by CNOP is stronger than that derived by LSV. CNOP acts as the optimal precursor of El Niño. (Xu, 2006).

El Niño is closely linked with the climatological mean state, the El Niño event itself and the initial error pattern. Analysis of the dynamic behavior of error growth for ENSO reveals the effect of nonlinear temperature advection on ENSO forecast uncertainties. The temperature advection process enhances the error growth for El Niño during AMJ. The larger the initial errors are, the greater the influence of the nonlinear temperature advection on spring error growth for El Niño events. In contrast, the impacts of nonlinearity on the error growth for La Niña events are negligible.

Based on the tendency equation of error growth, we demonstrate that the largest error growth of El Niño results from the collective of the climatological mean state, the structure of the El Niño anomaly itself and the patterns of the initial errors. Both the coupled ocean-atmosphere instability of climatological mean state and the dynamic instability of El Niño are the largest during boreal spring, which are most favorable for the largest CNOP growth of El Niño during boreal spring. The largest error growth of El Niño during spring is also closely associated with the pattern of the initial error. The enhancement of El Niño to CNOP growth could be understood as resulting from the same signs of the CNOP (initial error) for El Niño and the El Niño precursor. That is to say, if the initial error is not of the type of CNOP, the resultant ENSO prediction could be less uncertain.

Similar to the studies of ENSO precursors, considering the simplicity of the theoretical model, Xu (2006) also use CZ model to explore the seasonal dependence of error growth of ENSO warm event. It is shown that the error growth of El Niño during the growth phase is largest compared the other phases (for example, the mature phase and the decaying phase). This characteristics of El Niño error growth is investigated for different patterns of initial errors. The results demonstrate that CNOP has the most potential to show the dependence of error growth of El Niño on the growth phase of El Niño. Further analysis demonstrate that the CNOP-type initial error pattern of El Niño events are centralized locally over the equatorial central and

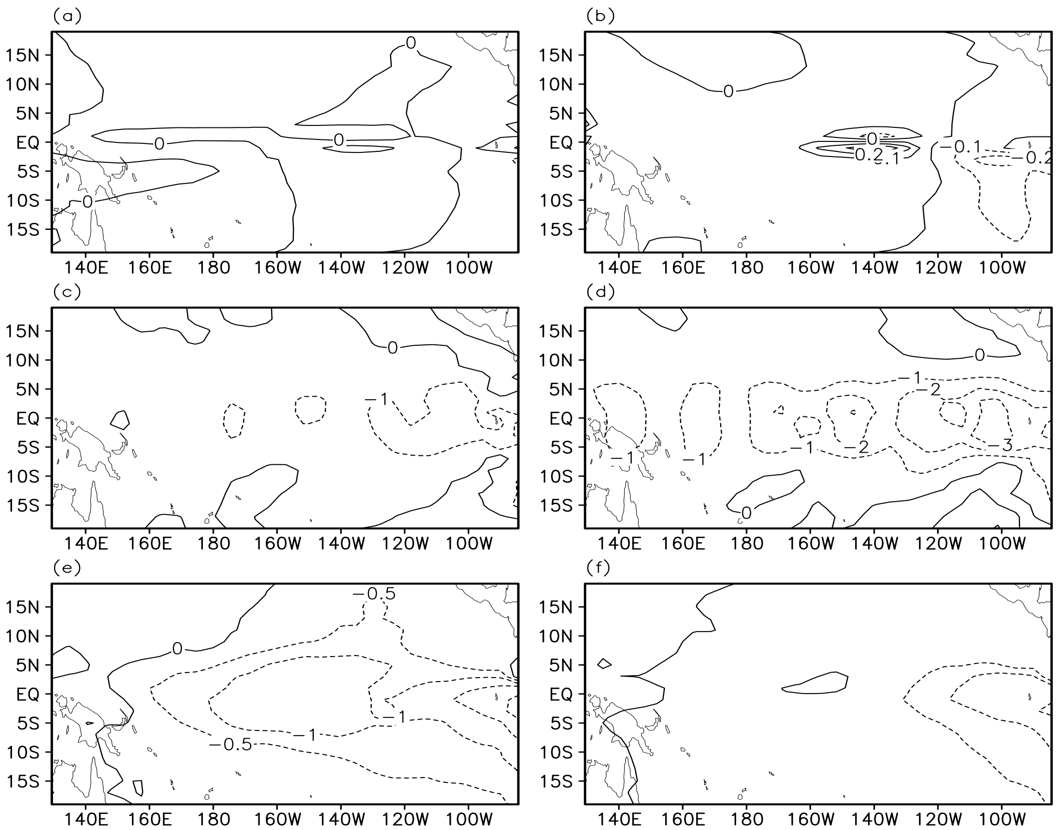

Fig. 4. Comparison between local CNOP and LSV. The left (right) column corresponds to local CNOP (LSV). (a), (c) and (e) are respectively the SSTA, thermocline depth anomaly, and the SSTA component of the nonlinear evolution for local CNOP. (b), (d) and (f) are those of LSV. The La Niña induced by local CNOP is stronger than that derived by LSV. local CNOP acts as the optimal precursor of La Niña. (Xu, 2006).

eastern Pacific. This indicates that the accuracy of the observation in this region may be important for reducing uncertainty of ENSO prediction. The above results emphasize that it is CNOP that induces the most prominent predictability barrier of ENSO. That is to say, if an initial error is not of the type of CNOP, the ENSO prediction may be less uncertain. Therefore, it is understandable why some authors reported the existence of SPB and Chen et al. (1995, 2004) pointed out that the predictability barrier for ENSO could be eliminated by improving the initial conditions. By the approach of CNOP, which allows us to exploit the importance of nonlinearity on SPB of ENSO, we have found the roles of the patterns of the initial error pattern, which suggests that if a data assimilation method could filter this kind of initial errors, ENSO predictability could be improved. Hence the above results can be considered as one of the theoretical bases for data assimilation of ENSO prediction. Besides, the locality of the CNOP-type initial error may encourage us to determine the most sensitive region of ENSO prediction by using CNOP approach, which belongs to the studies of targeting observation.

3.3 *Interdecadal change of ENSO asymmetry*

The observed El Niño events are generally stronger than the La Niña events. This property of ENSO is termed as ENSO asymmetry. Duan and Mu (2006) showed that this asymmetry has changed since the famous 1976 climate shift. Along the thinking of how the tropical background field modulates ENSO cycle, they explored the effect of the climatological basic-state change on the ENSO asymmetry by applying the approach of conditional nonlinear optimal perturbation (CNOP) in a theoretical coupled model. Observation shows that from the pre-shift (1961–1975) to the post-shift (1981–1995) period, significant changes have occurred in climatological background state, i.e., the mean temperature difference between the equatorial eastern and western Pacific basins and between the mixed-layer and subsurface-layer water, which control the ENSO oscillation in the theoretical coupled model. By computing the CNOPs of the climatological basic state corresponding to the 1961–1975 (1981–

1995) epoch, Duan and Mu (2006) reproduced the observed decadal change of ENSO asymmetry qualitatively. Based on the physics described by the model, the mechanism of ENSO asymmetry change in interdecadal scale was explored in depth. It is shown that the decadal change of ENSO asymmetry is induced by the change of nonlinear temperature advection, which is closely related to the decadal change of the tropical background state. These indicate that the decadal change of ENSO asymmetry results from the collective effect of the changes of the tropical background state and the nonlinearity. Duan and Mu (2006) therefore argued that the nonlinearity can explain not only the asymmetry of interannual ENSO, but also that of interdecadal ENSO, which may present a powerful evidence to the ENSO chaotic theory.

4. Applications of CNOP in the sensitivity analysis and the decadal variation studies of the ocean's THC

Ocean's THC plays an important role in climate variation, the studies of which is therefore received a great deal of attention, especially those of its sensitivity analysis to finite amplitude perturbation and decadal variation of THC. Although LSV can be used to investigate the sensitivity and the decadal variation of the flow, in the studies of sensitivity analysis, it cannot provide critical boundaries on finite amplitude stability and reveal the nonlinear mechanism of the decadal variation of THC. Furthermore, for a THC system with multiple equilibriums and internal oscillatory modes, its response to a finite amplitude perturbation is a difficult nonlinear problem. To reveal the effect of nonlinearity on the sensitivity and decadal variation of THC, Mu et al. (2004) and Sun et al. (2005) employed respectively CNOP to determine the nonlinear stability boundaries of linearly stable thermohaline flow states and to explore the mechanism of decadal variation of THC. In this section, we will summarized these works.

4.1 *Sensitivity and stability of THC to the finite amplitude initial perturbations*

With a simple two-box model of the thermohaline circulation, Mu et al. (2004) extended the results on linear optimal growth properties of perturbations on both thermal- and salinity-dominated thermohaline flows to the nonlinear case by using CNOP. It is shown that there is an asymmetric nonlinear response of these flows with respect to the sign of the finite amplitude freshwater perturbation. In Mu et al. (2004), the authors computed the CNOPs of a two-box model of the thermohaline circulation respectively with thermal-dominated stable steady states (TH states) and salinity-dominated stable steady states (SA states), and studied the nonlinear developments of the finite amplitude perturbations of these two stable steady states for fixed model parameters.

In the case of TH states, the extensive numerical results were performed. It is demonstrated that the initial saline and freshwater perturbations of oceans THC behave symmetrically with respect to the sign of steady flow rate in the corresponding TLM. In the nonlinear two-box model adopted in Mu et al. (2004), due to the effect of nonlinearity, the nonlinear evolution of the freshwater (saline) perturbations leads to a larger (smaller) amplitude than their linear counterparts. This indicates that the perturbations which move the system towards a bifurcation point will be more amplified through nonlinear mechanisms than perturbations that move the system away from a bifurcation point. The authors also demonstrate that for the CNOPs with small amplitude, the flow rate recovers to the steady climate state rapidly. For the CNOPs with large initial amplitude, it takes much longer time for the thermohaline circulation to recover to steady state. This is different from the results of a linear analysis, which demonstrates that CNOP can reveal the effect of nonlinearity on THC.

In the case of SA states, there are similar results to TH state, that is, the CNOP always moves the system towards the bifurcation point. The SA states have an asymmetry in the nonlinear amplification of disturbances, with larger amplitude for initial salinity perturbation. In Mu et al. (2004), the authors also paid attention to the sensitivity of THC along the bifurcation diagram. The authors firstly calculated the CNOPs of the model along the TH branch for the continuous changing fresh water forcing. The results demonstrate that with the parameter changing, the linearly stable TH state gradually transits from nonlinearly stable state to nonlinearly unstable state. It is easily derived that for each value of this model parameter, a critical value of initial perturbation amplitude must exist so that the TH state is nonlinearly unstable, which induces a transition of the system from the TH state to the SA state. This critical value acts as the nonlinearly stability threshold of the thermohaline flows. For the SA branch, there are similar results. For simplicity, the detailed description is not shown here.

4.2 *Passive mechanism of decadal variation of THC*

The mechanism of THC variation is an open problem for scientists. The transient amplification of initial perturbations is important for thermohaline variability. Within the above simple two-box model, Sun et al. (2005) used CNOP approach to study the decadal variation of THC. It is shown that THC has two different types of initial perturbations in the nonlinear regime.

One is the freshwater flux perturbation, which is the CNOP of THC and has stronger amplification. The other is salinity flux perturbation, whose amplification is weaker. Freshwater (salt) perturbations weaken (enhance) the mean circulation and hence weaken (enhance) the stability of THC. Sun et al. (2005) also investigated the passive variabilities of THC by superposing the initial perturbations to the thermohaline circulation. They found that the passive variabilities in this model were due to the nonnormal and nonlinear growth of initial perturbations. These variabilities, measured as recovering time of perturbations, can cause decadal variability of THC.

The studies of Sun et al. (2005) demonstrated that that CNOP approach is applicable to the investigation of the dependence of the THC sensitivity on the background climate state and is of potential application to the interpretation of past climate change linked to THC variations in nonlinear regime.

5. Instability and sensitivity analysis of grassland ecosystem

Wang (2006) used CNOP approach used to study the instability and stability of grassland ecosystem by a theoretical model of Zeng et al. (2004). It is found that for the moisture index, μ, there exist two critical values, μ_1 and μ_2, which bound an interval. There exist one unstable equilibrium state, and two stable equilibrium state, grassland and desert in this interval. An abrupt transition between grassland and desert occurs around μ_1. But how the transitions between the different equilibrium state occur with finite amplitude initial perturbations is still unknown. That's the focus of this study. The extensive numerical experiments show that for the moisture index, μ, being larger than the bifurcation point μ_1 ($\mu_1 < \mu_2$), there exist a CNOP and a local CNOP of grassland equilibrium state, which stand for the most unstable (or most sensitive) mode of grassland ecosystem. The behaviors of CNOP and the local CNOP, which are superposed on the basic state (the grassland equilibrium state), depend on the moisture indexes. In case that the moisture index is larger than μ_2, the CNOP and local CNOP are both converged to the grassland equilibrium state. When the moisture index is larger than μ_1 and less than μ_2, the behaviors of the evolutions of CNOPs are different from the above case. In this situation, with a large number of numerical experiments performed on CNOP, we obtain the critical value, δ_c, of the magnitudes of the initial perturbations, which distinguish the unstable and stable mode of grassland ecosystem for a given moisture index. As shown in Fig. 5, when the magnitude of CNOP is larger than δ_c, the ecosystem evolves to the desert equilibrium state with the CNOP being initial perturbation, while the local CNOP drives the ecosystem evolving back to the grassland equilibrium state, which suggests that the ecosystem becomes more and more unstable with the increasing amplitudes of initial perturbations. However, if the magnitude of initial perturbations is smaller than δ_c, the CNOP and local CNOP all drive the ecosystem recovering back to the grassland equilibrium state. When the magnitude of initial perturbation is equal to δ_c, with CNOP being the initial perturbation, the ecosystem evolves to a steady state, which is the unstable equilibrium state, while the local CNOP drives the ecosystem evolving back to the grassland equilibrium state. These results demonstrate the nonlinear characteristic of instability and sensitivity of grassland ecosystem to finite amplitude perturbations, and suggest the potential applicability of CNOP in the study of the impact of human activities on grassland ecosystem. These primary results encourage us to investigate deeply the transitions of different ecosystem states by CNOP.

6. Discussion

This paper reviews the conditional nonlinear optimal perturbation and its applications in predictability studies and sensitivity analysis for the 20 anniversary of the Chinese National Committee for WCRP. CNOP represents the initial perturbation that satisfies certain constraint condition and has largest nonlinear evolution at prediction time. In the studies of prediction uncertainties of weather and climate, CNOP describes the initial error that has largest impact on the prediction results. CNOP has two intrinsic characteristics: (1) CNOP is considerably different from LSV for the large initial perturbations or/and long optimization time intervals. (2) in some cases, there exist local CNOPs of clear physical physical meaning. These characteristics were also examined in a more realistic model, which further shows the inherence of the characteristics of CNOP and indicates the feasibility of CNOP in relatively complex models.

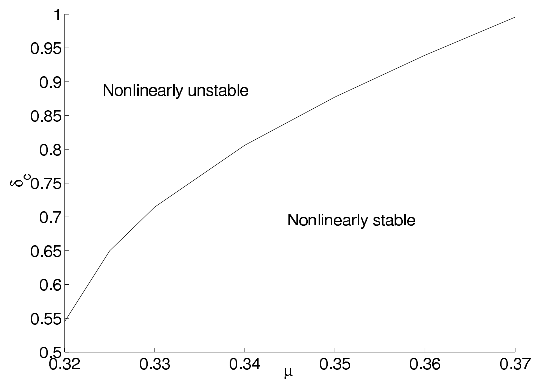

Fig. 5. The critical value of δ_c vs the parameter, moisture index, μ. This figure is from Wang (2006).

The applications of CNOP introduced in this paper include those of CNOP in ENSO predictability, and sensitivity and stability of thermohaline circulation and grassland ecosystem. The results demonstrate the applicability of CNOP in the studies of atmospheric and oceanic sciences and explore the physics of CNOP. The physics of CNOP depends on the particular physical problem. As the above is demonstrated by the authors and the collaborators, CNOP can describe the optimal precursor of certain weather or climate event. Also when the CNOP is considered to be the initial perturbation of certain weather or climate event, it stands for the initial error which has largest impacts on the uncertainty of the prediction. In sensitivity and stability analysis of fluid motion, CNOP describes the most unstable (or most sensitive) mode. All these suggest that CNOP is a proper approach to the predictability and sensitivity analysis. Besides, due to the nonlinearity of CNOP, it can be used to deal with the nonlinearity in predictability and sensitivity analysis. In the above exploration of predictability and sensitivity of weather and climate, this point of view is also be examined by the comparison of CNOP and LSV.

In calculating CNOP numerically, an efficient nonlinear optimization algorithm is essential, which guarantees the success of gaining CNOP. In this paper, for the low- and higher-dimensional model (maximal dimensions is 512), the sequential quadratic programme (SQP) algorithm or SPG2 solver has been proved to be successful for the nonlinearly constraint optimization problem. For the realistic forecast models, since they describe the intricate nonlinear atmospheric or oceanic flow motions and often have quite high dimensions, the nonlinear optimization problems involved could be difficult. Even in some cases, the problems are notsmooth. Nevertheless, the authors and their collaborators recently compute CNOPs of complex models, for examples, the coupled model of Zebiak and Cane (1987) and MM5 model. And the primary results suggest that CNOP can be calculated successfully. This is a great inspiritment. Simultaneously, encouraged by the work and the successful implemental of 4-dimensional variational data assimilations, it is expected that CNOP can be applied to more realistic models with quite high dimensions. Alternatively, inspired by the applications of linear singular vectors (LSVs) to the ensemble forecast at European Center for Medium-Range Weather Forecasts (ECMWF), the authors and the collaborators adopt CNOP to construct the initial fields of ensemble forecast studies and demonstrate primarily the feasibility of CNOP in ensemble forecast.

Predictability studies and sensitivity analysis are a field of challenge due to the nonlinearity and complexity of atmospheric and oceanic motions. However, it is expected that, with the development of computer and the in-depth collaboration of computational mathematician and meteorologist, significant progresses in the study of predictability and sensitivity analysis will be made in the future, in which CNOP approach is expected to play an important role.

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