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Numerical Weather Prediction: East Asian Perspectives

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Chapter 17
Conditional Nonlinear Optimal Perturbation: Applications to Ensemble Forecasting of High-Impact Weather Systems

Wansuo Duan, Lichao Yang, Zhizhen Xu, and Jing Chen

Abstract The conditional nonlinear optimal perturbation (CNOP) method, which includes CNOP-I for identifying the optimally growing initial perturbation, CNOP-P for revealing the most sensitive parameters, CNOP-B for disclosing the boundary uncertainty that exerts the largest effect on forecasts, and CNOP-F for exploring the combined effect of kinds of model errors, is introduced. Their applications to the ensemble forecasting of tropical cyclone and convectional scale weather systems are reviewed to show the usefulness of CNOP-I, -P, and -F in estimating the initial error effect, model parametric error effect, and even the combined effect of kinds of model errors, respectively. The future outlook and prospects are also provided.

Keywords Ensemble forecasting · Nonlinear optimal perturbation · Uncertainty · Tropical cyclone · Convectional scale weather systems

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17.1 Introduction

Ensemble forecasting is performed to evaluate forecasting uncertainties. Ensemble forecasting is often implemented by superimposing a group of mutually independent initial perturbations on the initial analysis of the control forecast to estimate the initial uncertainties and then the associated forecasting uncertainties (Leith 1974). Ensemble forecasting provides the ensemble mean forecasting results of concerned weather and climate events, the ensemble mean forecast error quantified by the ensemble spread, and probabilistic information about the occurrence of concerned events (Buizza et al. 2005; Bowler 2006; Leutbecher and Palmer 2008; Buckingham et al. 2010). Due to the diversity and usefulness of ensemble forecast products, ensemble forecasting is an irreplaceable method in numerical predictions. The World Meteorological Organization (WMO) has treated ensemble forecasting as one of the main development strategies of numerical predictions.

The traditional ensemble forecast, as introduced above, is applied to address the effect of initial uncertainties. Determining what kind of initial perturbations are more beneficial for estimating initial uncertainties and acquiring higher forecast skill is the essential issue of ensemble forecasting. Actually, the ensemble forecasting members are generated to neutralize the errors of the control forecasts and to make the members better characterize the true state. Since the errors of the control forecast often grow rapidly with time due to the instability of atmospheric and/or oceanic motions, a group of rapidly growing initial perturbations are expected to superimpose on the initial analysis of the control run to neutralize the forecast error growth. Thus, only rapidly growing initial perturbations can help improve the ensemble forecasting skill (Mureau et al. 1993; Toth and Kalnay 1993, 1997). Various approaches have been introduced to generate the growing-type initial perturbations for ensemble forecasting, and some of them have gained great success in operational weather forecasting and climate predictions. Toth and Kalnay (1993) developed the breeding method to identify growing-type initial perturbations, i.e., the bred vectors (BVs), and applied it to the ensemble forecasting system at the National Centers for Environmental Prediction (NCEP) in 1992. The European Centre for Medium-Range Weather Forecasts (ECMWF) introduced an alternative method of singular vectors (SVs; Mureau et al. 1993; Buizza and Palmer 1995; Molteni et al. 1996) and produced ensemble forecasts with great success. Ensemble forecasting, as mentioned above, requires growing-type initial perturbations superimposed on control forecasts to achieve higher forecasting skill; that is, it should be ensured that such initial perturbations grow rapidly during the forecast period. Notably, SVs possess clear dynamics to yield growing-type initial perturbations during the forecast period (Du et al. 2018). However, their fatal shortcoming is that they cannot cope with the impact of nonlinear physical processes on the amplification of the initial perturbations (Anderson 1997; Hamill et al. 2000; Mu 2000). To overcome this limitation, Mu et al. (2003) proposed the conditional nonlinear optimal perturbation (CNOP), which is an extension of the leading SV in the nonlinear regime. The CNOP fully considers the influence of nonlinear physical processes and represents the optimally growing initial perturbation in the nonlinear regime. Mu and Jiang (2008b) replaced the leading SV with the CNOP to produce ensemble initial perturbations and demonstrated higher forecast skills than SVs (also refer to Huo and Duan 2019; Zhou et al. 2021). To take into account fully nonlinear impacts in the development of the initial perturbations, Duan and Huo (2016) further formulated the orthogonal CNOPs (O-CNOPs) method to produce mutually independent nonlinear optimal initial perturbations for ensemble forecasting. The O-CNOPs have been shown to display a higher ensemble forecast skill than SVs and BVs and a more reasonable ensemble spread for estimating the uncertainty in a hierarchy of models (Duan and Huo 2016; Huo et al. 2019; Wang and Duan 2019; Wang 2021; Zhang et al. 2023a).

The ensemble forecasting methods mentioned above focus on addressing the initial uncertainty effects and are only reasonable under a perfect model assumption. For the model error effect, it is much difficult to estimate its uncertainties. Despite this disadvantage, some methods have been designed to address the corresponding forecasting uncertainties. For example, the ECMWF proposed a stochastically perturbed parameterization tendency scheme (SPPT; Buizza et al. 1999) and stochastic kinetic energy backscatter scheme (SKEB; Shutts 2005), leading to important improvements in the ensemble forecast skill (Du et al. 2018; also refer to the special issue, Buizza 2019). Hou et al. (2006) also developed a stochastic total tendency perturbation scheme (STTP) to emulate model uncertainties in the NCEP global ensemble forecasting system in February 2010 (also refer to Hou et al. 2008, 2010). As argued above, the ensemble forecasting system also requires growing-type perturbations to account for the unstable growth of forecast errors. However, the randomness of these model perturbation methods limits their ability to fully capture the rapid growth behavior of forecast errors caused by model errors. To obtain the rapidly growing model perturbations, Barkmeijer et al. (2003) proposed using a forcing singular vector (FSV) closely related to the SVs, which represents a rapidly growing constant tendency perturbation in a linear framework. This constant tendency perturbation describes the combined effects of the model systematic errors and parts of state-dependent model errors that are not explicitly described in the model equations (Feng and Duan 2013). To obtain a higher forecasting level, Duan and Zhou (2013) proposed approaching this problem by using the nonlinear forcing singular vector (NFSV). The NFSV is the tendency perturbation that implants the full nonlinear effect and makes the forecast deviate from the reference state more significantly. Relative to the CNOP method mentioned above, Wang et al. (2020b) also referred to the NFSV as CNOP-F, a special case of the CNOP, particularly for addressing the model error effect. If the NFSV is employed in an ensemble forecasting framework, it could better encompass the truth and provide more reliable ensemble members. To achieve this purpose, Duan et al. (2022a) proposed a new approach based on a set of orthogonal NFSVs (O-NFSVs), following the idea of O-CNOPs developed in Duan and Huo (2016). The O-NFSVs provide mutually independent model tendency perturbations that enable the description of the forecast uncertainties caused by model errors. Furthermore, Zhang et al. (2023b) are trying to apply O-NFSVs to yield model perturbations to imitate the model uncertainties responsible for tropical cyclone (TC) forecasts by using the realistic Weather Research and Forecast (WRF) model. Preliminary results showed the usefulness of O-NFSVs in offsetting the model error effect on TC forecasts.

To address the inevitable effects of both initial errors and model errors in numerical forecasts, Duan et al. (2022a) further developed C-NFSVs to combine all these uncertainty effects, which particularly consider the effect of initial and model error interactions and generalize the original NFSV only for measuring the model error effect, finally proposing a novel ensemble forecasting method. Although there have been ensemble forecasting systems that consider both initial error effects and model error effects (e.g., Buizza et al. 1999; Hou et al. 2010), they were built by superimposing the independent initial perturbations (such as SVs, BVs, or others) and the model tendency perturbations (e.g., SPPT or STTP). To date, no attention has been given to the dynamically coordinated growth of the initial and model perturbations, which may limit the skill of ensemble forecasts, and C-NFSVs may compensate for this gap.

In this chapter, we would summarize the advances in ensemble forecasting with respect to the implementation of the newly developed CNOP method and its applications to high-impact weather system forecasting. The subsequent section will introduce the idea of the CNOP method, and then Sect. 17.3 presents the applications to ensemble forecasting studies, especially for tropical cyclone and convectional scale weather systems. In Sect. 17.4, a novel ensemble forecasting method, C-NFSVs, to estimate the forecast uncertainties caused by both initial errors and model errors is introduced, and its special case, the O-NFSVs is described, accompanied by its applications to TC forecasts. Finally, a summary and prospect are provided in Sect. 17.5.

17.2 Conditional Nonlinear Optimal Perturbation

Since Mu et al. (2003) proposed the CNOP method (also refer to Mu and Duan 2003), it has been extended from the original CNOP representing the optimally growing initial errors (for convenience, hereafter CNOP-I; Mu et al. 2003; Mu et al. 2010) to additional CNOP-P for addressing the influences of optimally growing model parametric errors (Mu et al. 2010), CNOP-B for disclosing the boundary uncertainties that have the largest effect on forecasts (Wang and Mu 2015), and CNOP-F [i.e., the nonlinear forcing singular vector proposed in Duan and Zhou (2013)] for exploring the combined effect of various model errors. This section focuses on introducing the ideas of CNOP-I, -P, and -F, which have been applied in ensemble forecasting studies. The specifics are presented as follows.

The dynamic equations responsible for atmospheric and oceanic motions are generally written as a nonlinear partial differential equation (Eq. (17.1)).

$$\begin{cases} \dfrac{\partial U}{\partial t} = F(U, P) \\ U|_{t=0} = U_0 \end{cases} \quad \text{in } \Omega \times [0, T], \tag{17.1}$$

where $U(x, t) = [U_1(x, t), U_2(x, t), \ldots, U_n(x, t)]$ is the state vector, U_0 is its initial value, F is the nonlinear differential operator, $\mathbf{x} = (x_1, x_2, \ldots, x_n), t \leq T(0 < T < +\infty)$ is the time, $P = (P_1, P_2, ..., P_m)$ are model parameters, P_i represents one model parameter independent of time *t,* and Ω is a domain in the *n*-dimensional Euclidean space $\mathbb{R}^n$. Since a forecast is often contaminated by initial and model errors, Eq. (17.1) is rewritten as Eq. (17.2) to represent forecast model equations consisting of both initial perturbation u_0 and parametric perturbation p.

$$\begin{cases} \dfrac{\partial (U + u)}{\partial t} = F(U + u, P + p) \\ U + u|_{t=0} = U_0 + u_0 \end{cases} \quad \text{in } \Omega \times [0, T], \tag{17.2}$$

where u represents the departure from the reference state U caused by the combined effects of initial and parametric perturbations. In this circumstance, when a nonlinear optimization problem is defined as in Eq. (17.3), its solution (u_0^*, p^*) represents the optimal combined mode of initial and parametric perturbations that satisfies a certain constraint and results in the largest departure from the reference state at time τ. Mu et al. (2010) referred to this combined mode as CNOP.

$$J(u_0^*, p^*) = \max_{(u_0, p) \in C_{u_0, f}} \|u(\tau)\|, \tag{17.3}$$

where C confines the scope of the initial perturbation u_0 and parametric perturbation p. The CNOP has two special cases: the first case is CNOP-I, proposed by Mu et al. (2003), which is used to reveal the optimally growing initial perturbation when the parametric perturbation $p = 0$, while the second case is CNOP-P, which causes the largest departure from the reference state when the initial perturbation disappears (Mu et al. 2010).

If one rewrites Eq. (17.2) as Eq. (17.4), its resultant forecast can be understood as being influenced by the combined effect of the initial error and the model errors contained in the total tendency. $\tilde{F}$ associated with the uncertainties in the sub-grid process parameterization, the external forcing and the stochastic noises, and other kinds of model uncertainties.

$$\begin{cases} \dfrac{\partial (U + u)}{\partial t} = \tilde{F}(U + u) \\ U + u|_{t=0} = U_0 + u_0 \end{cases} \quad \text{in } \Omega \times [0, T]. \tag{17.4}$$

If the total tendency $\tilde{F}$ is rewritten into two terms F^0 and f and F^0 represents the accurate tendency, then the term f represents the total tendency error. It is obvious that the tendency error f is composed of different kinds of model errors. In this sense, Eq. (17.4) can be expressed as Eq. (17.5).

$$\begin{cases} \dfrac{\partial(U+u)}{\partial t} = F^0(U+u) + f(x,t) \\ U+u|_{t=0} = U_0+u_0 \end{cases} \quad \text{in } \Omega \times [0,T]. \tag{17.5}$$

Then, which tendency error will cause the largest forecast error at the forecast time when the initial errors are neglected? Eq. (17.6) would produce such optimal tendency error, or more generally, optimal tendency perturbation f^*.

$$J(f^*) = \max_{f \in C_f} \|u(\tau)\|. \tag{17.6}$$

This optimal tendency perturbation is the nonlinear forcing singular vector (NFSV) proposed by Duan and Zhou (2013). Relative to the CNOP, Wang et al. (2020b) took the NFSV as a special case of the CNOP and denoted it as CNOP-F, particularly for exploring the combined effect of different kinds of model errors.

Thus, a family of CNOPs has been achieved, including CNOP-I, -P, -F introduced above and CNOP-B formulated for exploring the boundary condition error that has the largest effect on the forecasting results by Wang et al. (2020b). All these perturbations fully considered the effects of nonlinear physical processes and have been shown to represent the optimally growing mode in their respective scenarios. One can search for the CNOPs by using existing optimization solvers such as Spectral Projected Gradient 2 (SPG2; Birgin et al. 2000) or Limited memory Broyden–Fletcher–Goldfarb–Shanno for bound-constrained optimization (LBFGS-B; Liu and Nocedal 1989) according to the descending direction provided by the gradients of relevant cost functions. Notably, some intelligent algorithms, such as particle swarm optimization (PSO) and genetic algorithms, have emerged to solve similar optimization problems. These algorithms do not calculate the gradient and may be applicable to models with different complexities.

The CNOPs have been widely applied to reveal the sensitivity and uncertainties of atmospheric and oceanic motions and to address associated problems of target observations, data assimilation and ensemble forecasting of high-impact weather and climate events, such as TCs, atmospheric blocking, El Niño-Southern Oscillation (ENSO), the Indian Ocean Dipole (IOD), atmospheric environmental heavy air pollution and oceanic mesoscale eddies (Duan et al. 2022b). In this chapter, we would summarize the advances in ensemble forecasting studies for high-impact weather systems.

17.3 Applications to Ensemble Forecasting for High-Impact Weather Systems

In the applications of CNOPs, CNOP-I was applied to ensemble forecasting for addressing effect of initial uncertainties, and O-CNOPs were proposed to produce mutually independent ensemble initial perturbations as requested by ensemble forecasting (refer to the introduction; Duan and Huo 2016); subsequently, the O-CNOPs were applied to forecast TC tracks and achieved higher forecasting skills than the BVs and SVs methods (Huo and Duan 2019; Huo et al. 2019). For the model error effect, the CNOP-P, which, as introduced in Sect. 17.2, solves the optimal parametric perturbation, was employed in the ensemble forecasting of convective-scale weather systems based on its recognized sensitivity to parameter uncertainties (Wang et al. 2020a). Furthermore, considering that CNOP-P only accounts for the effect of model parameter errors and other kinds of model errors and that their interactions also substantially disturb weather and climate predictions, Xu et al. (2022a) adopted the NFSV [also referred to as CNOP-F in Wang et al. (2020b)] to measure the combined effect of various model errors to explore ensemble forecasting in convective-scale weather systems [also refer to Xu et al. 2022b]. Both CNOP-P and CNOP-F achieved much higher forecasting skills than the operational use of SPPT. In this section, we will provide a thorough overview of all CNOP applications to ensemble forecasts of high-impact weather systems.

17.3.1 Forecasts of Tropical Cyclone Events Associated with the Initial Error Effect

For ensemble forecasting, to consider the effect of nonlinearity on ensemble initial perturbations, Mu and Jiang (2008a, b) introduced the CNOP method to SV ensemble forecasting by replacing the leading SV with the CNOP (also refer to Jiang and Mu 2009) and attempted to improve the related ensemble prediction skill. However, such an approach still involves linear approximation because nonleading SVs still have the role of ensemble initial perturbations. Inspired by this limitation, Duan and Huo (2016) developed O-CNOPs based on CNOP-I by applying Eq. (17.1).

$$J(u_{0j}^{*}) = \max_{\mathbf{u}_{0j} \in C_{u_{0j}}} \left\| u_{j,\tau} \right\|, \tag{17.7}$$

where

$$C_{u_{0j}} = \begin{cases} \left\{ u_{0j} \in \mathbb{R}^n \middle| \left\| u_{0j} \right\| \le \delta \right\}, j = 1 \\ \left\{ u_{0j} \in \mathbb{R}^n \middle| \left\| u_{0j} \right\| \le \delta, u_{0j} \perp C_{u_{0k}}, k = 1, \ldots, j-1 \right\}, j > 1 \end{cases}, \tag{17.8}$$

$C_{u_{0j}}$ is one subspace of the whole phase space, $\mathbb{R}^n$ is the n-dimensional Euclidean space, "$\perp$" is the orthogonality of vector spaces, u_{0j} is the initial perturbation in $C_{u_{0j}}$, and $\|u_{0j}\| \leq \delta$ is the constraint condition (δ is a positive constant); then, u_{0j}^* is the j-th CNOP-I. According to Eqs. (17.7) and (17.8), the first CNOP-I (i.e., u_{01}^*) possesses the largest nonlinear evolution in the first subspace, i.e., the whole space, and the j-th CNOP-I (i.e., u_{0j}^*) possesses the largest nonlinear evolution in the subspace orthogonal to the $j-1$ CNOP-Is (i.e., $u_{01}^*, u_{02}^*, \ldots, u_{0j-1}^*$). These CNOP-Is constitute the O-CNOPs. The O-CNOPs are all derived from nonlinear model and fully consider the effect of nonlinearity (Duan and Huo 2016).

Duan and Huo (2016) first used the famous Lorenz-96 model (Lorenz 1996) to test the dynamics of O-CNOPs. They found that when the initial analysis errors are fast-growing, the ensemble forecasts generated by O-CNOPs perform much more skillfully; however, for the slowly growing initial analysis errors, the ensemble forecasts generated by O-CNOPs achieve almost the same forecast skill as those generated by SVs when the ensemble initial perturbations are sufficiently small, whereas the ensemble forecasts generated by SVs possess higher skill when the ensemble initial perturbations are much larger. The authors also showed that the O-CNOPs are more applicable than the SVs for achieving much higher ensemble forecast skill of extreme events. In particular, the ensemble forecasts generated by the O-CNOPs require a very small number of ensemble members to achieve high forecast skills. Therefore, the O-CNOPs may provide another useful technique to generate initial perturbations for ensemble forecasting.

Huo et al. (2019) further applied the O-CNOPs to the realistic Pennsylvania State University/National Center for Atmospheric Research (PSU/NCAR) Fifth-Generation Mesoscale Model (MM5; Dudhia 1993; Grell et al. 1994) for the ensemble forecasts of five TC tracks; furthermore, they made a thorough skill comparison with the SVs since they present different patterns from O-CNOPs [see Fig. 17.1, for the STY Matsa (2005)]. The results showed that the ensemble members generated by the O-CNOPs present large spreads but tend to be located on the two sides of real TC tracks and show agreement between ensemble spreads and ensemble mean forecast errors for TC tracks. This finding indicates that these members generated by the O-CNOPs are more feasible to reveal forecast uncertainties of TC tracks than SVs in terms of these TCs. The results also illustrated that the O-CNOPs of smaller amplitudes are more reasonable to construct the ensemble members for short lead–time forecasts but that those of larger amplitudes should be utilized for longer lead–time forecasts due to the stronger effects of nonlinearities. In particular, Huo and Duan (2019) compared O-CNOPs to the ensemble strategy of replacing the leading SV with the first CNOP-I (hereafter CNOP + SVs), as in Mu and Jiang (2008b), in an attempt to reveal the importance of the nonlinear effect in yielding ensemble initial perturbations. The authors showed that the CNOP + SV ensemble strategy is not necessary to produce greater ensemble forecast skill than that of the SVs, but it is certain that the O-CNOPs are more likely to cause much higher ensemble forecasting skill for TC tracks. These results indicate that the inclusion of fully nonlinear effects on ensemble initial perturbations enhances the ensemble forecasting skill for TC tracks.

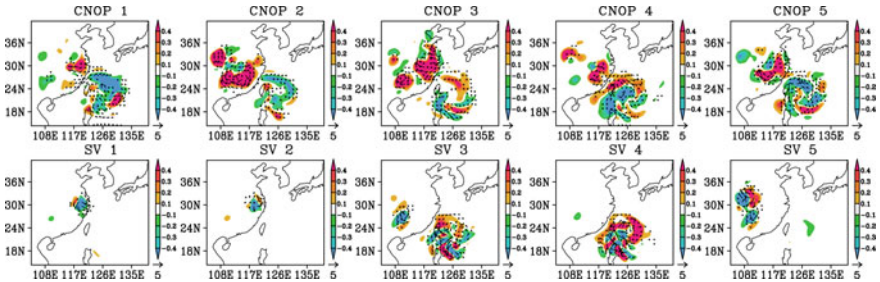

Fig. 17.1 Spatial structures of the temperature (shaded) and wind components (vectors) of the first five O-CNOPs and SVs for the STY Matsa (2005) at the level σ = 0.975, which were used to produce the ensemble perturbations. The columns list the structures in sequence, from the first to the fifth. From Huo and Duan (2019). ©2019 Springer Nature Publisher. Used with permission

With the more advanced WRF model, Zhang et al. (2023a) predicted another twelve strong TC cases by using the O-CNOPs ensemble forecasting method, and as expected, obtained much higher ensemble forecasting skill compared with the SVs, BVs, and random perturbations (RPs) methods (Fig. 17.2). In particular, the authors demonstrated that the ensemble members generated by the O-CNOPs are more likely than those made by BVs, SVs, and RPs to reproduce the tracks of unusual TCs, such as the sharp northward-turning track of Megi (1013) and the counterclockwise loop track of Tembin (1214). Zhang et al. (2023a) also showed that when the WRF was compared with MM5, its resultant ensemble forecasts made by the O-CNOPs still significantly increased the forecasting skill of the TC tracks in the control forecasts, although the control forecasts possessed a higher skill than MM5, while those generated by SVs and BVs only slightly improved it or became much worse. That is to say, the SVs and BVs are less functional in improving the track forecasting skill in the much more advanced WRF model for the twelve selected TCs.

Based on the above findings, O-CNOPs are shown to be useful in estimating initial uncertainties and then yielding much higher ensemble forecasting skill from the realistic MM5 to the advanced WRF model for TC track forecasting. However, the results are still derived from a small number of TC cases and therefore are only indicative. Despite this finding, the results have made us confident in validating the usefulness by using more TC cases, even to apply O-CNOPs to the real-time forecasts of TCs. Other high-impact weather systems, even high-impact climate events, should also be used to investigate the effectiveness of O-CNOPs. Then, operational suggestions can be provided.

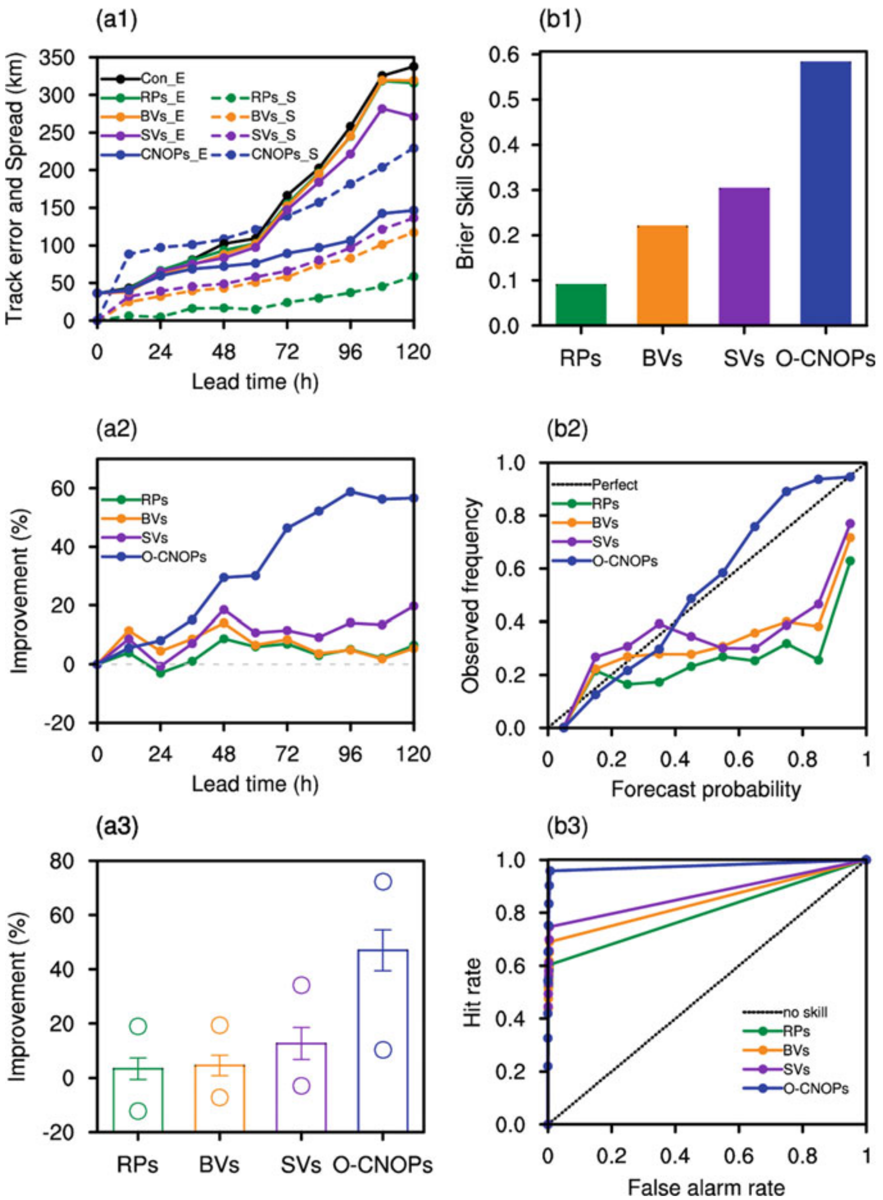

Fig. 17.2 **a1** Track errors (solid lines) at different forecast times for the control forecast and ensemble mean forecasts averaged for twelve selected TCs and the corresponding ensemble spreads (dotted lines) generated by the RPs (green), orthogonal BVs (yellow), SVs (purple) and O-CNOPs (blue); **a2** the error reduction rate (i.e., skill improvement) due to the ensemble mean; and **a3** the box-and-whisker plot for the skill improvement averaged for twelve TCs and all lead times, with a 95% confidence interval. The circles denote the maximum and minimum of the improvements for the twelve TCs; **b1**, **b2**, and **b3** plot the Brier skill score, reliability diagram, and relative operating characteristic curve for the TC strike probability, respectively, generated by the four methods

17.3.2 Forecasts of the Convectional Scale Weather System Associated with the Model Error Effect

It is worth noting that convection-allowing ensemble prediction systems with high resolutions of 2–4 km have emerged as a major focus and become a hot topic of current research on numerical weather predictions. How to accurately address model uncertainties in a convective-scale system is a crucial issue in studies of convection-scale ensemble forecasts. Even if ensemble techniques have been applied, the ensemble members generated by the stochastically perturbed physics tendencies (SPPT; Buizza et al. 1999) utilized in operational centers still face new scientific challenges, especially the problem of under-dispersion. To address this under-dispersion issue, Wang et al. (2020a) applied the CNOP-P approach to the Global and Regional Assimilation and Prediction Enhanced System (GRAPES), which is a convection-scale ensemble prediction model, to detect the most sensitive parameters. Then, the authors formulated a kind of parameter perturbation by adding a group of stochastic perturbations to these sensitive parameters to depict the model uncertainty and conducted ensemble forecast experiments on relevant variables at convective scales. The authors showed that the relevant ensemble members, compared with those generated by the SPPT, enable a much larger spread for humidity and temperature over the troposphere and yield much more reliable forecasting skill on near-surface variables and precipitation. This study concludes that the application of the CNOP-P sensitivity to identifying parametric uncertainties greatly improves the ensemble forecasting skill of convectional scale weather systems, even to a higher skill than the SPPT employed in operational centers.

It is easily recognized that the CNOP-P only accounts for the effect of model parameter errors, and other kinds of model errors also severely influence weather and climate predictions; in particular, these model errors are interactive. Considering this situation, Xu et al. (2022a) further adopted the CNOP-F [i.e., the NFSV in Duan and Zhou (2013)] to measure the combined effect of various model errors to explore the ensemble forecast of convective scales [also refer to Xu et al. 2022b]. The authors superimposed the structured NFSV on the SPPT perturbations and formulated new tendency perturbations (denoted by "SPPT_NFSV") for ensemble forecasts. With these new perturbations, Xu et al. (2022b) conducted ensemble experiments by using the GRAPES convection-scale ensemble prediction model adopted in Wang et al. (2020a). The authors illustrated that the overall probabilistic skills are obviously improved by using the SPPT_NFSV and have an advantage over the SPPT (Fig. 17.3). Particularly, the authors demonstrated that the use of the NFSV enhances the forecasting skill of precipitation accuracy. It is inferred that additional structured nonlinear perturbations (e.g., the NFSV) superimposed on the SPPT can better represent model uncertainties in convection-scale ensemble forecasts and finally contribute to a more comprehensive characterization of model uncertainties for convective-scale forecasts.

Either the CNOP-P or the NFSV provides more sensitivity information related to the model perturbations and thus leads to higher ensemble forecasting skill of

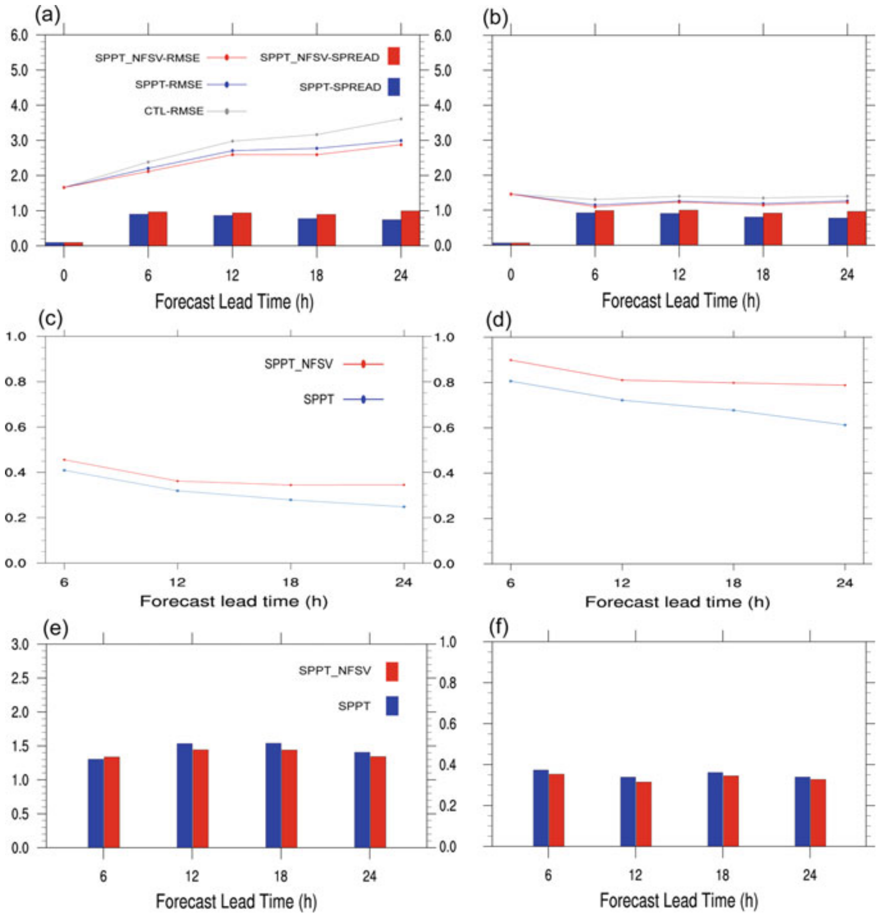

Fig. 17.3 Probabilistic skill for 500 hPa zonal wind (left column) and temperature (right column). **a** and **b** show the domain-averaged RMSE of the control forecast (gray line), SPPT_NFSV experiment (red line), and SPPT experiment (blue line), with the ensemble spread for the SPPT_NFSV (red bar) and SPPT (blue bar). **c** and **d** represent the spread-error consistency, **e** and **f** depict the continuous ranked probability score, **g** and **h** illustrate the Talagrand rank histograms, and **i** and **j** indicate the outlier scores. From Xu et al. (2022b). @ John Wiley and Sons Publisher. Used with permission

the convectional scale system by reasonably enlarging the ensemble spread. This finding indicates that the errors of the control forecasts caused by model errors in the GRAPES often grow at a fast rate and that the model perturbations related to CNOP-P and NFSV provide growth behavior that is much closer to the dynamical growth of the model errors than the SPPT provides. Then, it is naturally questioned whether the model perturbation strategies implemented as above are most applicable for capturing the rapid growth behavior of the forecast errors induced by the model errors. In particular, for the NFSV, if the SPPTs are fully replaced by mutually

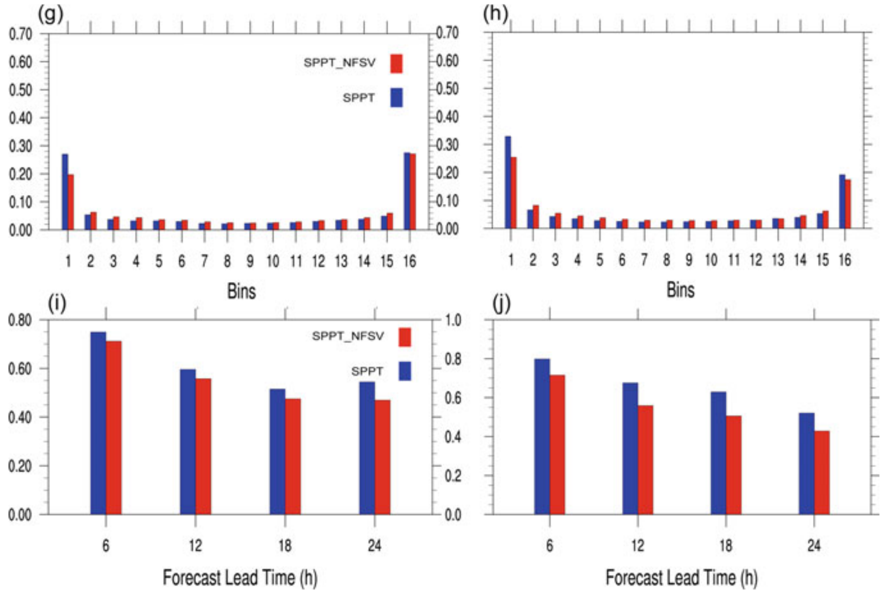

Fig. 17.3 (continued)

independent NFSVs, similar to O-CNOPs, can they depict much better the model uncertainties that exhibit fast growth behaviors and obtain much higher forecast skill? All these questions deserve our in-depth investigations. It is expected that a more efficient and user-friendly ensemble forecasting method for addressing the model error effect, even the combined effect of model and initial errors, will be investigated.

17.4 A Novel Ensemble Forecasting Method for Addressing the Combined Effect of Initial and Model Errors and Its Special Case O-NFSVs Accompanied by Applications to TC Forecasts

The ensemble forecasting mentioned in Sect. 17.3 focuses on considering either the initial uncertainties or model uncertainties. However, in realistic forecasting systems, the effects of both the initial errors and the model errors, especially the effects of their interaction, are inevitable (Nicolis et al. 2009). In this actual situation, the key question is how to correctly combine the initial errors and model errors to obtain reliable ensemble forecasting. As discussed in the introduction, although there exist ensemble forecasting systems that consider the combined effect of initial and model errors (e.g., Buizza et al. 1999; Hou et al. 2010), they yielded initial perturbations (such as SVs and BVs) only for measuring initial uncertainties and model perturbations (e.g., SPPT or STTP) merely for estimating model uncertainties and did not consider the dynamically coordinated growth of initial and model perturbations, which may limit the skill of ensemble forecasting.

Duan et al. (2022a) generalized the original NFSV (also CNOP-F; refer to Sect. 17.2) for measuring model error effects and proposed C-NFSVs that combine the impacts of both model errors and initial errors, formulating a novel ensemble forecasting method that considers the dynamically coordinated growth of initial and model perturbations. The specific equations are Eqs. (17.9) and (17.10).

$$J(\boldsymbol{f}_j^*) = \max_{\boldsymbol{f}_j \in C_j} \left\| u_\tau(r\boldsymbol{f}_j; \boldsymbol{f}_j) \right\|_b, \tag{17.9}$$

where

$$C_j = \begin{cases} \boldsymbol{f}_1 \in \mathbb{R}^n, \left\| \boldsymbol{f}_1 \right\|_a \le \sigma_f, \\ \left\{ \boldsymbol{f}_j \in \mathbb{R}^n \middle| \left\| \boldsymbol{f}_j \right\|_a \le \sigma_f, \boldsymbol{f}_j \perp C_k, k = 1, \ldots, j-1 \right\}, j > 1, \end{cases} \tag{17.10}$$

and $\left\| r\boldsymbol{f}_j \right\|_a \le \sigma_I$; $r\boldsymbol{f}_j$ is the initial perturbation with $r = \frac{\sigma_I}{\sigma_f}$ and $\boldsymbol{f}_j$ is the tendency perturbation used to offset the model errors in Eq. (17.4) [i.e., tendency errors $f(x, t)$ in Eq. (17.5)], $\mathbb{R}^n$ is the n-dimensional Euclidean space, the symbol $\{\cdot\}$ refers to an ensemble of vectors, and the symbol $\perp$ indicates the orthogonality; $\|\cdot\|_a$ and $\|\cdot\|_b$ are the norms that are applied to measure the amplitudes of the initial perturbations $r\boldsymbol{f}_j$ and tendency perturbations $\boldsymbol{f}_j$ and the departure from the reference state at time τ, respectively; σ_f and σ_I are positive constant numbers that constrain the amplitudes of the tendency perturbations and initial perturbations, respectively. The combined modes $(r\boldsymbol{f}_j^*, \boldsymbol{f}_j^*)$ are defined as the C-NFSVs.

The C-NFSVs have two particular cases: O-CNOPs and O-NFSVs. The former has been proposed in Duan and Huo (2016) to address the initial error impact on the forecasts (refer to Sect. 3.1), while the latter estimates the model error impact, as proposed by Duan et al. (2022a). Duan et al. (2022a) adopted the famous Lorenz-96 model and demonstrated that ensemble forecasting based on O-CNOPs has a higher skill than that based on O-NFSVs in the early stage of the forecasts, while in the later stage of the forecast, the impact of the model errors becomes more prominent and ensemble forecasting based on O-NFSVs excels. The forecasts based on C-NFSVs, due to their optimization on both the initial errors and model errors, possess higher skill than those based on O-CNOPs and O-NFSVs during the whole forecast period. In Duan et al. (2022a), a simple combination of O-CNOPs and O-NFSVs was also compared with C-NFSVs; the results showed that the simple combination may cause inconsistent dynamical behaviors between the initial perturbations and the tendency perturbations and would degrade the ensemble forecasting skill, while the C-NFSVs possess additional dynamical features that lead to a higher forecast skill (Fig. 17.4). These results justify the advantage of using C-NFSVs in building ensemble forecasts.

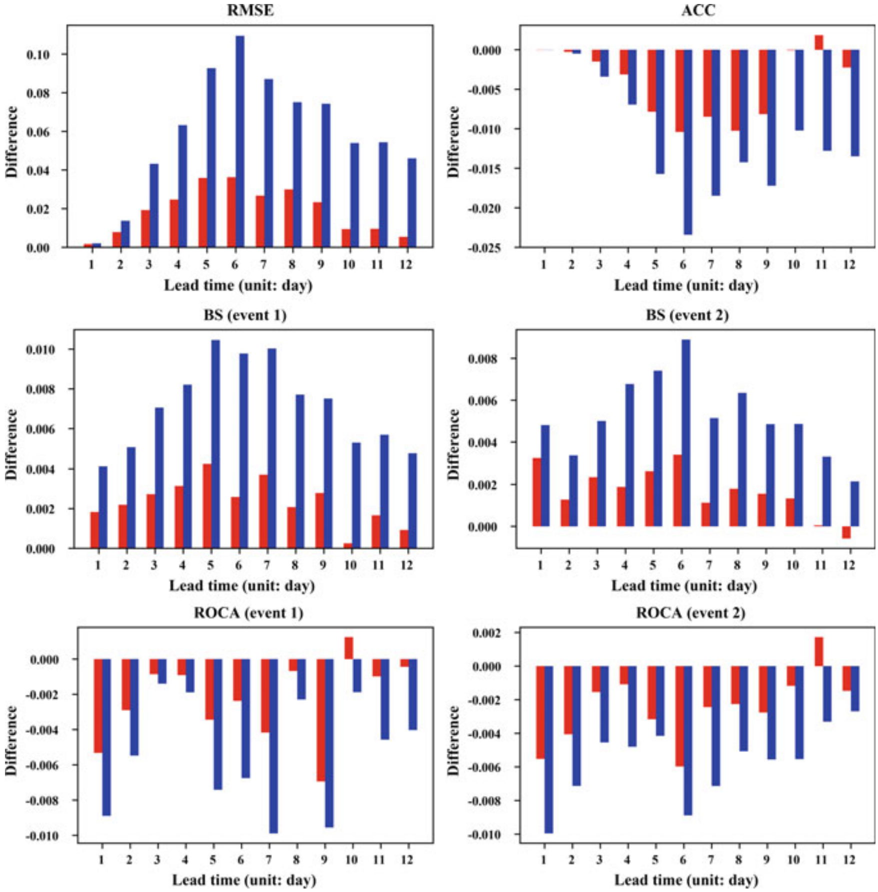

Fig. 17.4 Skill performance differences between the ensemble forecasts made by the combination of O-CNOPs and O-NFSVs when they obtain the highest skill scores and those made by C-NFSVs (red) and the skill performance differences between the combination of O-CNOPs and O-NFSVs with the same optimization period and perturbation amplitudes as in C-NFSVs and those made by C-NFSVs (blue). The horizontal axis denotes the lead time, and the vertical axis represents the differences in the RMSE, ACC, BS, and ROCA values. From Duan et al. (2022a). ©2022 American Meteorological Society. Used with permission

Considering that TC track forecasts have considerably improved during the past decades while the TC intensity forecasts remain challenging, Zhang et al. (2023b) adopted the special case of C-NFSVs, i.e., O-NFSVs for estimating the effect of model uncertainties to conduct TC ensemble forecasting experiments with the WRF model, with a focus on improving TC intensity forecasting skill (Fig. 17.5). The authors performed a comparison between the O-NFSVs and the traditional SKEB and SPPT schemes. The results demonstrated that the O-NFSV ensembles provide a better representation of the model uncertainties affecting TC intensification, with much better deterministic and probabilistic skills. Similar improvements were also extended to the forecasting skill for TC tracks, although the perturbations were not optimized for that specific purpose. Therefore, O-NFSVs could be a kind of perturbation structure that is able to describe the uncertainties in TC intensity and tracks. Furthermore, Zhang et al. (2023b) showed that such perturbations are also favorable for recognizing a TC's rapid intensification process during forecast periods.

Although the O-NFSV structures presented in Zhang et al. (2023b) are realized using mutually independent, growing-type tendency perturbations to represent model uncertainties for TC forecasts, ensemble forecasting to address the combined effect of initial and model errors has not yet been implemented. Therefore, a natural extension of this study will be conducted to explore the application of the C-NFSVs in TC forecasts. It is expected that when the O-NFSVs are properly combined with initial perturbations by the C-NFSVs, new highly reliable ensembles will be available

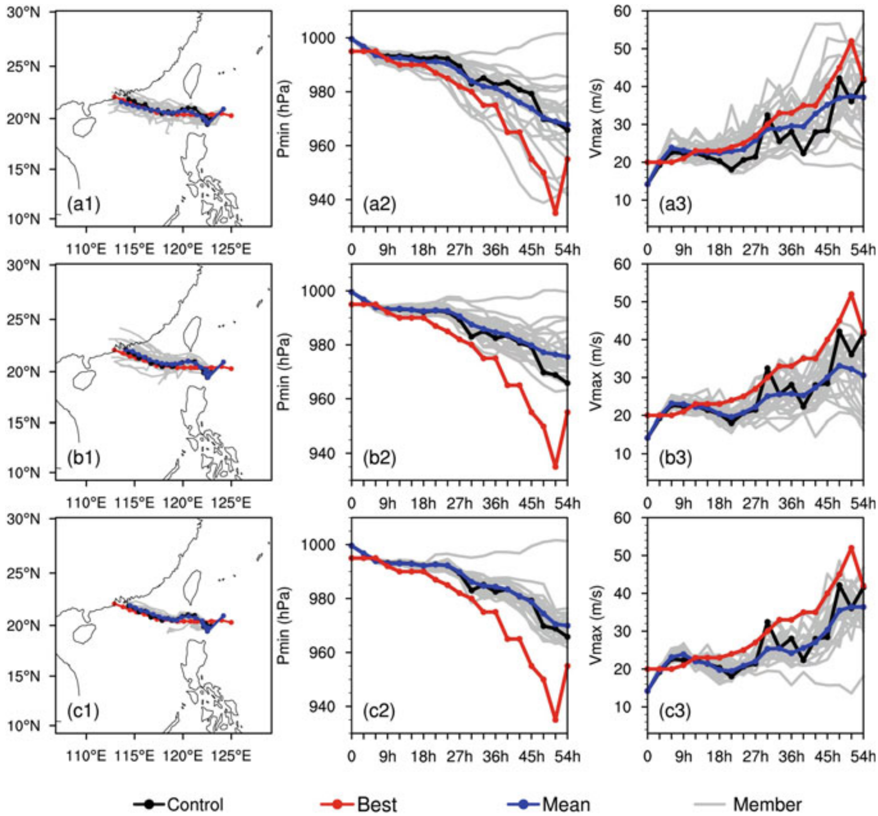

Fig. 17.5 Ensemble forecasts generated by O-NFSVs **(a)**, SKEB **(b)** and SPPT **(c)** for track (1), Pmin (2), and Vmax (3) of TC Hato (201713). The black lines represent the control forecasts, the red lines denote the best tracks, the blue lines indicate the ensemble means, and the gray lines represent the ensemble members

for further improving TC forecasting skill in terms of intensity and track, even the much more challenging TC precipitations. For the convectional scale weather prediction systems investigated above, it is also anticipated that the ensemble forecasting experiments using the C-NFSVs can be conducted to examine if they, compared with those using the NFSV_SPPT in Sect. 17.3, perform much well in depicting model uncertainties and achieving higher forecasting skill.

17.5 Summary and Prospect

Studies on the CNOP method and their applications to ensemble forecasting for high-impact weather systems are summarized. The CNOP method which includes CNOP-I for revealing the optimally growing initial perturbation, CNOP-P for extracting the most sensitive parameters, CNOP-B for determining the boundary condition uncertainty that exerts the largest effect on forecasts, and CNOP-F for exploring the combined effect of kinds of model errors, is introduced. These CNOPs fully consider the effect of nonlinearity and provide a way to obtain the optimally growing-type error mode for predictability studies, including ensemble forecasting in their respective scenarios.

The CNOP-I, -P, and -F [i.e., NFSV] have been applied to the ensemble forecasting of high-impact weather systems. O-CNOPs were proposed to produce ensemble perturbations for estimating the initial uncertainties, and with the applications to the numerical models from the conceptual Lorenz-96 model to the realistic MM5 and further to the advanced WRF, they were demonstrated to have the ability to represent the initial error effect and to promote the ensemble forecasting skill. Especially for the TC track forecasts, O-CNOPs, compared with the operationally utilized SVs and BVs, provide the ensemble members that exhibit larger spreads but tend to be located on the two sides of real TC tracks and show much better agreement between ensemble spreads and ensemble mean forecast errors. Furthermore, O-CNOPs were illustrated to be favorable for reproducing the unusual TC tracks in forecasts. For the TC intensity forecasts, the O-NFSVs developed from the CNOP-F were used to depict the effect of model errors, and it was revealed that the ensemble members generated by the O-NFSVs have the ability to represent the model uncertainties affecting TC intensification and to provide much higher ensemble forecasting skills than the operationally employed SPPT and SKEB. This ability was also extended to the forecasting skill for TC tracks, although the O-NFSVs were not optimized for that specific purpose. For convectional scale weather systems, ensemble forecasting focuses on addressing the model uncertainty effect. An alternative perspective has been applied to provide the relevant sensitivities using both CNOP-P and CNOP-F, which have helped extract more sensitive ensembles or exert more unstable model perturbations on the SPPT ensembles, consequently promoting the ensemble forecasting skill of convectional scale weather systems to a higher level. To explore an ensemble forecast method to address the combined effect of initial and model errors, the C-NFSV perturbation scheme was proposed, and its particular feature of dynamically coordinated growth of initial and model perturbations was found to be responsible for the ensemble forecasting skill being higher than any simple combination between initial perturbations and model perturbations. It is hoped that the C-NFSVs can be continually applied in realistic forecasts for high-impact weather systems and further increase the forecast skill by reasonably estimating the combined effect of initial and model errors. In particular, comparisons between C-NFSVs and the combined modes of other types of initial perturbations (e.g., BVs or SVs) and other types of model perturbations (such as SPPT or SKEB) are worth performing in the future. Another interesting avenue in the development of C-NFSVs is to consider the effect of time-varying stochastic errors; a combined mode of C-NFSVs and random forcing tendency perturbations may cover a broader range of model errors and have potential for further improving the ensemble forecast skill.

Either O-CNOPs for addressing initial uncertainties, O-NFSVs for interpreting the model uncertainties or C-NFSVs that combine initial and model error effects are worthy of further investigation, especially in applications to realistic models with different complexities for high-impact weather event and even high-impact climate event forecasting. Furthermore, it is noted that these perturbations are the optimally growing mode in nonlinear models in their respective scenarios; thus, it is naturally questionable whether associated ensemble forecasts prefer to achieve high skill in the forecasts of extreme events [also refer to Duan and Huo (2016)]. These will be the subjects of follow-up work. In addition, computational efficiency is a challenge of any ensemble forecast method; whether the presently popular machine learning algorithm can be combined with ensemble forecasting to save time and increase the efficiency of ensemble forecasting should also be explored. In any case, with the development of computing sciences and emerging disciplines and technologies, the above ensemble forecast methods would become much useful and applicable in realistic forecasts of high-impact weather and climate events.

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