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Conditional Nonlinear Optimal Perturbation: A New Approach to the Stability and Sensitivity Studies in Geophysical Fluid Dynamics

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Abstract

In the stability, sensitivity and predictability studies in geophysical fluid dynamics, linear singular vector (LSV), which is the fastest growing perturbation of the linearized model, is one of the useful tools. However, the linear approximation has strong limitations on the applicability of LSV, since it ignores the nonlinear processes, such as wave-mean flow interactions. The authors have proposed a new method called CNOPs (Conditional Nonlinear Optimal Perturbations), which generalizes LSV into the fully nonlinear category. CNOP is the initial perturbation whose nonlinear evolution attains the maximum value of the cost function, which is constructed according to the problems of interests with physical constraint conditions. In sensitivity and stability analysis of fluid motions, CNOP describes the most unstable (or most sensitive) initial modes. It can also represent the optimal precursor of certain weather or climate event, or stand for the initial error that has largest effect on the uncertainties at the prediction time.

In this review paper, we introduce the concept of CNOPs first. Then we present the results on the stability, sensitivity and predictability obtained by CNOP approach, which includes: the sensitivity and stability of ocean's thermohaline circulation; predictability of El Nino-Southern Oscillation; nonlinear stability problems of a theoretical grassland ecosystem model. It is shown that CNOPs not only reveal the effect of nonlinearity on the physical problems in which nonlinear process plays an important role, but also demonstrate significant physical characteristics that cannot be shown by LSV. For example, in Zebiak-Cane model, CNOPs, rather than LSVs, act as the initial anomaly patterns that evolve into ENSO events most probably, which shows that nonlinearity enhances the evolution of El Nino. In the theoretical Stommel's model, a nonlinear asymmetric response of THC to the finite perturbation is revealed by using CNOP approach, which cannot be realized by LSV.

Other applications of CNOP, which includes ensemble forecast and target observations, are reviewed too. Prospect and challenge in the future applications of CNOP are also discussed.

Introduction

One of the central problems in the studies of geophysical fluid dynamics (GFD) is the stability and instability of atmospheric and oceanic motions, which has attracted numerous distinguished scientists. Quite a few methods have been developed from theoretical, experimental to numerical approaches.

For linear stability studies in GFD, linear singular vector (LSV), which was introduced into GFD by Lorenz in 1960's, has been widely used. Since LSV describes the fastest growing perturbation of a linearized model, it is usually used to find the most unstable mode of atmospheric or oceanic flows [9]. Recently, its applications have been extended to explore climate variability and predictability [26, 31]. However, LSV is established on the basis that the evolutions of initial perturbations can be governed approximately by linearized models, which, due to the absence of nonlinearity, cannot describe the nonlinear evolution of the finite amplitude perturbations. The atmospheric and oceanic motions are generally dominated by nonlinear systems, so the nonlinearity limits the applicability of LSV.

Realizing the limitations of LSV, Oortwijn and Barkmeijer [27] and Barkmejier [1] applied a modification technique to the linearly fastest-growing perturbation and constructed fast-growing perturbations for the nonlinear regime. However, they also recognized that this technique may not necessarily result in the nonlinearly fastest growing perturbations. Mu et al. [16] proposed a new method called conditional nonlinear optimal perturbations (CNOPs), which is a kind of initial perturbations that satisfy a certain physical constraint and has the largest nonlinear evolution at a future time. Then this method has been used to explore the studies of sensitivity, stability, and predictability of weather and climate [6, 7, 8, 10, 16, 17, 19, 20, 22, 23, and 25]. All these works showed that CNOPs usually capture the nonlinear effects on the perturbation evolutions of our interested problem and some new conclusions are found by this approach.

Considering the potential applicability of CNOPs to fluid dynamics, but almost all papers related to CNOP have been published in journals of atmospheric or oceanic researches, which might be ignored by the scientists in the field of fluid dynamics, the purpose of this review paper is to introduce the approach of CNOP to fluid dynamical researchers. The main results, which are obtained by the authors and their colleagues with CNOP approach, are summarised. In next section, we will first review the approach of CNOPs.

Conditional nonlinear optimal perturbation

Suppose that the fluid motions are governed by the following nonlinear system:

$$\begin{cases} \dfrac{\partial w}{\partial t} + F(w) = 0, \\ w|_{t=0} = w_0, \end{cases} \tag{2.1}$$

where the state vector

$$w(x,t) = (w_1(x,t), w_2(x,t), \cdots, w_n(x,t)),$$

$x = (x_1, x_2 \cdots, x_n)$ are the spacial variables, t is time, and $(x,t) \in \Omega \times [0,T], \Omega$ a domain in R^n, $T < +\infty$. The operator F in Eq. (2.1) is a nonlinear partial differential operator, and w_0 is the initial state. We assume that the future state of the fluid motions can be determined by integrating Eq. (2.1) with the appropriate initial condition. The solution to Eq. (2.1) for the state vector w at time τ is given by

$$w(x,\tau) = M_\tau(w_0) \tag{2.2}$$

Here M_τ is the propagator, which, as described by (2.2), "propagates" the initial value to the time τ in the future. In

GFD, its discrete form is usually called a "model". Let $U(x,t)$ and $U(x,t)+u(x,t)$ be the solutions of Eq (2.1) with initial value U_0 and U_0+u_0, respectively, where u_0 is an initial perturbation. We have

$$U(\tau)=M_\tau(U_0),\quad U(\tau)+u(\tau)=M_\tau(U_0+u_0).$$

So $u(\tau)$ describes the evolution of the initial perturbation u_0.

For a chosen norm $\|\cdot\|$, an initial perturbation $u_{0\delta}$ is called CNOP under the constraint $\|u_0\|\le\delta$, if and only if

$$J(u_{0\delta})=\max_{\|u_0\|\le\delta} J(u_0) \qquad (2.3)$$

where

$$J(u_0)=G(M_\tau(U_0+u_0)-M_\tau(U_0)) \qquad (2.4)$$

The functional $G(\cdot)$ is a measurement, which measures the evolution of the perturbations and particularly can be a norm ($\|\cdot\|$) of the state variables. Usually the constraint condition is simply expressed as belonging to a ball with the chosen norm. Obviously, we can also investigate the situation that the initial perturbations belong to other kind of functional set. Furthermore, the constraint condition could be some physical laws that initial perturbation should satisfy.

CNOP is the global maximum of $J(u_0)$ in the ball $\|u_0\|\le\delta$. It is also possible that there exist local maximum value of $J(u_0)$. In this case, we call the corresponding maximum as a local CNOP, denoted by $u_{0\delta}^l$.

CNOPs have clear physical and dynamical meanings. First, in the studies of sensitivity and stability analysis of fluid motions, it describes the most unstable (or most sensitive) initial perturbation of the fluid motions within the given finite time period. Second, when CNOP is considered to be an initial perturbation superposed on a weather or climate event, it is the initial error that has the largest effect on the prediction uncertainty. Third, when the initial perturbations are the initial anomalies, the CNOP represents the optimal precursor of certain climate or weather events. Finally, CNOP can also be used to estimate the upper bound of the prediction error [18]. Assuming U_0 is an initial observation, and U_T^t is the true value of the state, then the prediction error is

$$E=\left\|M_\tau(U_0)-U_T^t\right\|$$

If the propagator M_τ is considered to be exact, the prediction error E at prediction time T is only caused by the initial observational error. Since the true value of the state cannot be obtained exactly, it is impossible to get the exact value of E (prediction error). But if we know some information on the errors of the initial observation, e.g., the initial observation error in terms of a norm is less than δ, we can estimate the prediction error,

$$E_u=\max_{\|u_0\|\le\delta}\left\|M_\tau(U_0+u_0)-M_\tau(U_0)\right\|$$

where u_0 is the initial perturbation superimposed on initial observation U_0 and satisfies the constraint condition $\|u_0\|\le\delta$. Obviously, the inequality $E\le E_u$ holds. E_u gives the upper bound of the prediction error [18], whose expression is the same as $J(u_{0\delta})$ in (2.3) with U_0 being initial observation. Thus, CNOP gives the upper bound of the prediction error caused by initial uncertainties satisfying the constraint condition.

Based on these above physical explanations of CNOP, CNOP has been applied to study a variety of problems in GFD. In the following, we mainly review the applications of CNOP in the sensitivity and stability studies of ocean's thermohaline circulation (THC) and the predictability studies of El Nino-Southern Oscillation (ENSO). Some other applications including ensemble forecast, target observation, and the sensitivity of grassland ecosystem are simply introduced. With these applications of CNOP, we summarize the characteristics of CNOP and tell the algorithms that compute CNOP. Finally, the challenge and prospect in the applications of CNOP are discussed.

Sensitivity and stability of THC to finite amplitude perturbations

What is THC? It is known that the water of the great world ocean is constantly in motion, which carries colossal amounts of water to transport around the globe and has been named "The Great Ocean Conveyor" [4]. The THC is what drives the Conveyor. It involves both heat, hence "thermo", and salt, hence "haline". The circulation of heat and salt through the ocean basins is called the THC. Since the transport of both salt and heat is quite advection dominated and both quantities are not mixed well once deep below the surface, a particular amount of water can be traced back to its origin. Hence, such a volume of water can be characterized by its temperature and salinity at formation and is called a water mass. The two attributes, temperature and salinity, determine the density of seawater, whose differences in density between the water masses in the world's oceans causes the water to flow. The THC thereby produces the greatest oceanic current and works in a fashion similar to a conveyor belt -- hence the name -- transporting enormous volumes of cold, salty water from the North Atlantic to the Northern Pacific, and bringing warmer, fresher water in return.

One of the fundamental issues on climate variability is the stability and sensitivity of the ocean's THC. The sensitivity of the thermohaline circulation is caused by several feedbacks induced by the physical processes that determine the evolution of the thermohaline flow. One of these feedbacks is the salt-advection feedback, which is caused by the fact that salt is transported by the thermohaline flow, but in turn influences the density difference that drives this flow. The salt advection feedback can be conceptually understood in a two-box model [28], where it is shown to cause multiple equilibria and hysteresis behavior. Knutti and Stocker [11] investigated the sensitivity of the THC to perturbations. It is found that this sensitivity severely limits the predictability of the future THC when approaching the bifurcation point. Although LSV can be used to investigate the stability and sensitivity of the flow [17], it cannot provide critical boundaries on finite amplitude stability of the THC. Furthermore, due to the linearity of LSV, it cannot be used to study the response of a THC system with multiple equilibriums and internal oscillatory modes to a finite amplitude perturbation.

To investigate the effect of nonlinear processes on the sensitivity of THC, Mu et al [17] employed CNOP to determine the nonlinear instability boundaries of linearly stable thermohaline flow states by the famous Stommel's two-box model of the ocean's THC [28]. There are two steady states of the THC in the Stommel's model. One is called thermally-driven

(TH) state; that is, a negative equatorial-to-pole temperature gradient exists dominating the density. The other is called salinity driven (SA) state; that is a negative equatorial-to-pole salinity gradient exists dominating the density. Mu et al. [17] extended the results on linear optimal growth properties of perturbations on both TH and SA thermohaline flows to the nonlinear case.

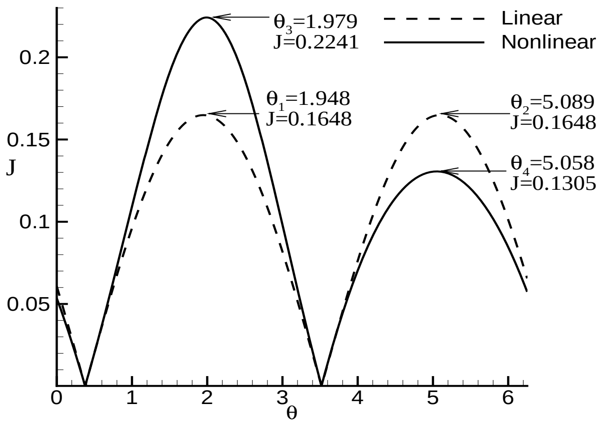

Figure 1. The asymmetric nonlinear response of THC to initial freshwater perturbation. The solid line (dash line) represents the nonlinear (linear) evolution J of the initial perturbation. The nonlinearity makes the evolution of the freshwater perturbation much faster. This figure is from [17].

In the case of TH states, the CNOPs superimposed on the TH steady stable state were calculated. All these CNOPs locate the boundaries of the constraint. The evolutions of the CNOPs were also investigated. The numerical results demonstrated that the initial saline and freshwater perturbations of ocean's THC behave symmetrically with respect to the sign of steady flow rate in the linearized Stommel's model (Figure 1). However, in the nonlinear Stommel's model, due to the effect of nonlinearity, the nonlinear evolution of the freshwater (saline) perturbations leads to a larger (smaller) amplitude than their linear counterparts (Figure 1), which indicates that the perturbations which move the system towards a bifurcation point will be more amplified through nonlinear mechanisms than perturbations that move the system away from a bifurcation point. The authors of [17] also demonstrate that for the CNOPs with small amplitude, the flow rate recovers to the steady climate state rapidly. For the CNOPs with large initial amplitude, it takes much longer for the THC to recover to steady state. This is different from the results of a linear analysis and reflects the effect of nonlinearity on THC.

In the case of SA states, there are similar results to TH state, that is, the CNOP always moves the system towards the bifurcation point. The SA states have an asymmetry in the nonlinear amplification of disturbances, with larger amplitude for initial salinity perturbation.

In [17], the authors also paid attention to the sensitivity of THC along the bifurcation diagram. The results demonstrate that with the parameter changing, the linearly stable TH state gradually transits from nonlinearly stable state to nonlinearly unstable one (Figure 2). It is easily derived that for each value of the model parameter, a critical value of initial perturbation amplitude must exist such that the TH state is nonlinearly unstable, which induces a transition of the system from the TH state to the SA state (Figure 3). This critical value acts as the nonlinearly stability threshold of the thermohaline flows. For the salinity-driven branch, the nonlinearly instability thresholds were also provided based on the Stommel's model.

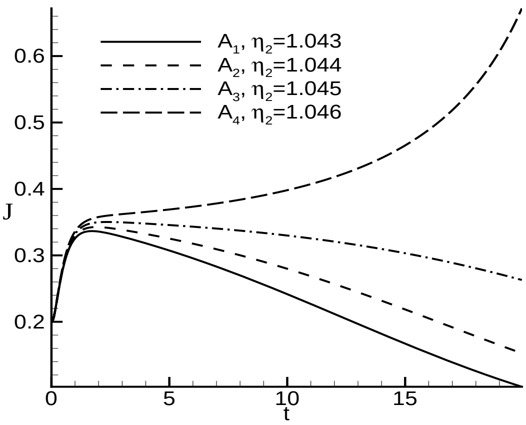

Figure 2. The nonlinear evolutions J of CNOPs with different values of freshwater parameter η_2. When $\eta_2 = 1.046$, the finite amplitude perturbation causes the transition of climate equilibrium. This figure is from [17].

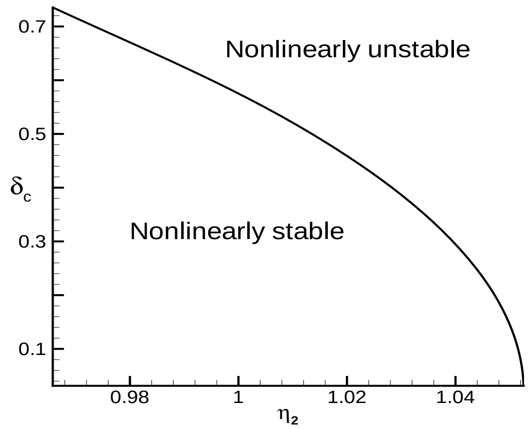

Figure 3. The critical value δ_c (of initial perturbation amplitude) for nonlinear stability versus the parameter controlling the thermally-driven state near the saddle-node bifurcation at $\eta_2 = 1.05$. This figure is from [17].

Within the above two-box model, Sun et al. [30] further studied the decadal variation of THC. They found that there are two different types of optimal perturbation in the nonlinear regime. One is the freshwater flux perturbation, which is the CNOP of THC and has stronger amplification. The other is salinity flux perturbation, whose amplification is weaker. Freshwater perturbations weaken the mean circulation and hence weaken the stability of THC, while salt perturbations enhance the mean circulation and the stability of THC. By superimposing the initial perturbations to the thermohaline circulation, Sun et al. [30] investigated the passive variabilities of THC. It is found that the passive variabilities are due to the nonnormal and nonlinear growth of initial perturbations. These variabilities, measured as recovering time of perturbations, can cause decadal variability of THC.

Recently by using CNOP, Wu and Mu [32] studied the impact of wind-driven ocean gyres (WDOG) on THC within a modified Stommel's box model, in which WDOG is considered in the form of diffusion [14]. Focuses are on the multi-equilibriums existence and nonlinear stability of THC. They reported that there exists a physical mechanism, which makes WDOG result in more decrement of salinity difference in the meridional direction than that of temperature difference. Consequently the effects of WDOG on the thermally and salinity driven equilibriums of THC are different: WDOG stabilizes the former but destabilizes the latter.

Wu and Mu [33] also investigated the impact of WDOG on the decadal variability of THC. Numerical analyses demonstrates that the physical mechanism found in Wu and Mu [32] plays different roles on the decadal variability of thermally and salinity driven equilibriums of THC too.

Applications of CNOP to ENSO predictability

Predictability studies in numerical weather and climate are concerned with the uncertainties of prediction caused by both initial and model errors. Clearly unstable states and chaotic process amplify the initial errors and finally lead the failure of predication. This implies that predictability studies are essentially related to the stability and sensitivity studies in atmospheric and oceanic sciences. One approach to attack the predictability problems is to investigate the evolution of the initial uncertainties that has the largest impact on the uncertainties of prediction [13]. CNOP acts as the initial uncertainties that have the largest negative effect on the prediction results. The authors and their colleagues used CNOP to explore ENSO predictability.

ENSO is a well-known short-term climate phenomenon that happening in tropical Pacific Ocean. Usually the annual mean seas surface temperature (SST) over the equatorial Pacific takes on a strong asymmetry between the relatively warm western part of the basin, the region is called the warm pool, and the cooler eastern basin, called the cold tongue. In some years, the SST anomaly of the equatorial eastern Pacific is up to a few degrees, the phenomenon of which is usually as El Nino. This phenomenon is associated with the atmosphere, and thus the term ENSO that incorporates the Southern Oscillation phenomenon is commonly used. Southern Oscillation refers to a seesaw shift in surface air pressure at Darwin, Australia and the Southern Pacific Island of Tahiti. Though they originate in the tropical Pacific, they have an impact on weather and climate globally and can cause natural disasters.

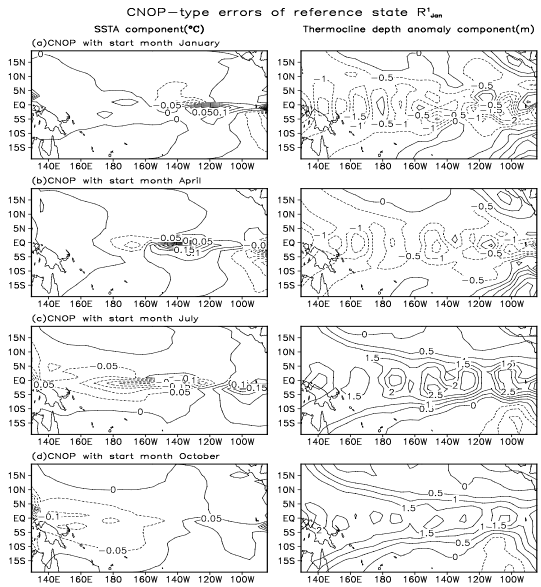

Figure 4. The patterns of CNOP-type error for a given El Nino event in the ZC model. (left) SSTA and (right) thermocline depth anomaly components for the start month being (a) January, (b) April, (c) July, and (d) October. This figure is from [20].

Since later 1980's, considerable efforts have been devoted to study and to predict the ENSO phenomenon. An important aspect of these studies is on the exploration of "spring predictability barrier" (SPB) for ENSO. SPB is a well-known characteristic of ENSO forecasts, which is referred to a phenomenon that most ENSO prediction models often experience an apparent drop in prediction skill across April and May [36]. Many works have investigated this phenomenon [26, 34, 35, 36, etc.]. It is noticeable that Chen et al. [5] reported that by using the initial field produced by a data assimilation approach, SPB in the model of Zebiak-Cane model (ZC model; [37]) is not as severe as that in persistence or in most other forecast models, which indicates the importance of the accuracy of initial fields in ENSO predictability.

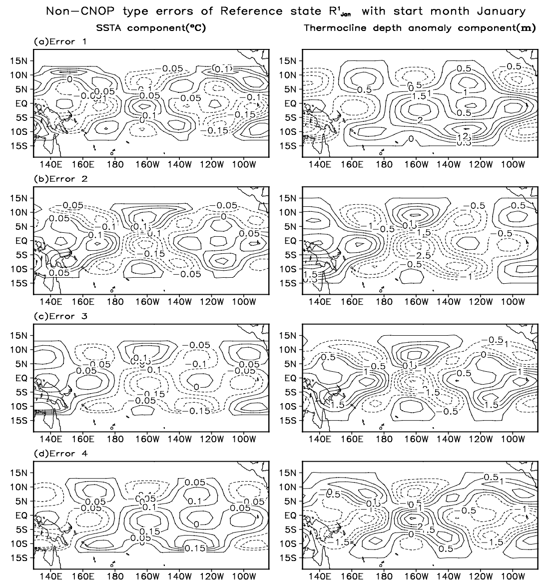

Figure 5. Four representatives of Non-CNOP type errors for the El Nino events as in Figure 4, where the start month of prediction is January. (left) SSTA and (right) thermocline depth anomaly components. This figure is from [20].

Encouraged by the work of Chen et al. [5], Mu et al [20] investigated SPB problem for ENSO events in the ZC model, in view of the development of initial errors, by using CNOP approach, where CNOP is superimposed on the ENSO events and plays the role of the initial error that has largest negative effect on the prediction results. In [20], CNOPs were calculated for different types of El Nino events with different start months. Then the CNOP-type error patterns were obtained (Figure 4). Integrating the ZC model with these initial errors, the dynamics of the corresponding prediction errors was illustrated. By investigating the slope of the prediction error evolution at different seasons, Mu et al. [20] reported that CNOP-type error tends to have a significant season-dependent evolution with the largest error growth occurring in northern spring, and produces most considerable negative effects on the forecast results, that is, the largest prediction error. Therefore, CNOPs are closely related to SPB. On the other hand, some other kinds of initial errors (Figure 5), whose patterns are different from those of CNOPs, have also been found. Although the magnitudes of such initial errors are the same as those of CNOPs in terms of the chosen norm, they either show less prominent season-dependent evolutions, or have trivial effect on the forecast results, and consequently do not yield SPB. The results of this investigation suggest that the CNOP-type errors can be considered as one of candidate errors that cause the SPB. If data assimilation or (and) targeting observation approaches possess the function of filtering the CNOP-type or (and) other similar errors, it is hopeful to improve the prediction skill of ENSO.

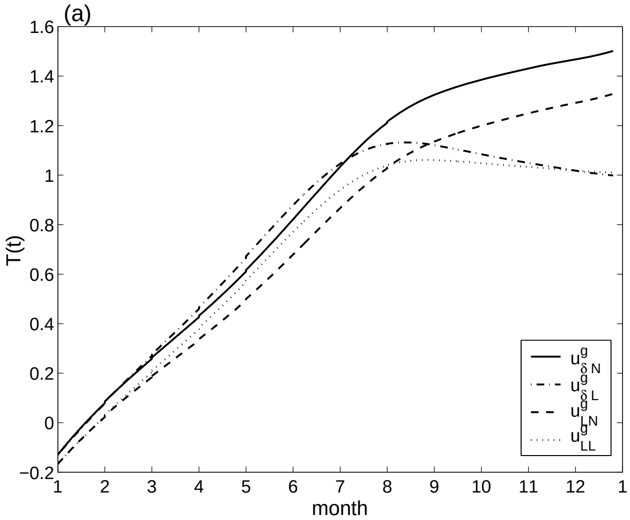

Figure 6. Nonlinear and linear evolutions of the nondimensional model variable T (SSTA) corresponding to CNOP and LSV of annual cycle, respectively. $u^g_{\delta N}\left(u^g_{LN}\right)$ and $u^g_{\delta L}\left(u^g_{LL}\right)$: the nonlinear and linear evolutions of SSTA of the CNOP (the LSV). This figure is from [6].

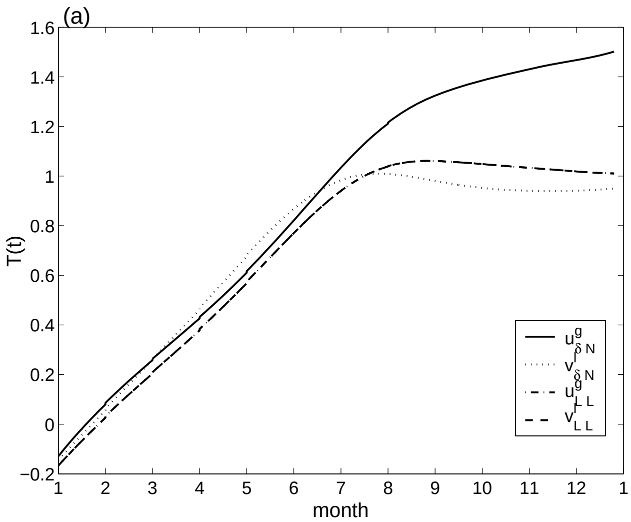

Figure 7. Comparisons between El Niño and La Nina events. $u^g_{\delta N}\left(u^l_{LL}\right)$: the SSTA nonlinear (linear) evolution of the CNOP (the corresponding LSV), and $v^l_{\delta N}\left(v^l_{LL}\right)$: the SSTA nonlinear (linear) evolution of $-T$ (negative anomaly of SST) of the local CNOP (the corresponding LSV). This figure is from [6].

Optimal precursor is another important problem of predictability studies for ENSO. The optimal precursor for ENSO is the initial anomaly mode that evolves into ENSO event most probably, which is the most predictable, meaning that if this signal related to CNOP is observed in nature, then the future outcome of the system is fairly certain. By using CNOP approach, Duan et al. [6] studied the optimal precursor problem for ENSO events by a simple coupled ocean-atmosphere model, where the CNOP is superimposed on the climatological annual cycle and represents an anomaly signal of ENSO events. For the different start months, the CNOPs were computed [6]. By studying the behaviour of the evolutions of these CNOPs, Duan et al. [6] demonstrated that the CNOPs of the climatological annual cycle evolves into the positive SST anomaly in the nonlinear model, which takes a striking resemblance to the development of El Nino (Figure 6). In fact, it acts as a precursor for El Niño event in the adopted model. Although the corresponding LSV also develops into an El Niño, the intensity is considerably weaker than that of CNOP. In this sense, Duan et al. [6] regarded CNOP as the optimal precursor for El Niño. For the local CNOP of the annual cycle, its nonlinear SST anomaly evolution is only a little larger than that of the corresponding LSV. This phenomenon can be explained by the locality of the local CNOP. As to the physical characteristic local CNOP bears, by investigating the nonlinear evolution, it is found that local CNOP acts as the optimal precursor of the model La Nina event. Furthermore, Duan et al. [6] compared the amplitude of El Nino and La Nina events in the adopted model. The theoretical ENSO events show themselves the significant asymmetry in amplitude (Figure 7), which originates the nonlinear feedback: the nonlinearity enhances the El Nino, but not La Nina event. These theoretical results were verified by the 22-year NCEP reanalysis data qualitatively.

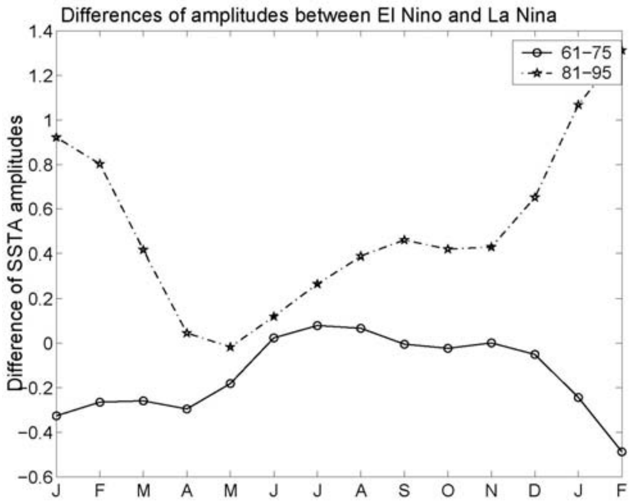

Figure 8. Difference between the composite El Nino and La Nina in term of SSTA amplitude. The line 61-75 (81-95) illustrates the amplitude difference between El Nino and La Nina during 1961-1975 (1981-1995). The values denoted by the line 81-95 are larger than those denoted by the line 61-75, showing that the asymmetry of the observed El Nino and La Nina during 1981-1995 is stronger than that during 1961-1975. This figure is from [7].

The observed El Nino events are generally stronger than the La Nina events, which is termed as ENSO amplitude asymmetry in literatures. The theoretical results related to CNOP also revealed this property of ENSO (see the paragraphs related to optimal precursor for ENSO). Duan and Mu [7] demonstrated that this asymmetry has changed since the famous 1976 climate shift (Figure 8). Along the thinking of how the tropical background field modulates ENSO cycle, they explored the effect of the climatological basic-state change on the ENSO asymmetry by applying the CNOP approach in a theoretical coupled model. Duan and Mu [7] found that from the preshift (1961–1975) to the postshift (1981–1995) period, significant changes have occurred in the observed climatological background state, i.e., the mean temperature difference between the equatorial eastern and western Pacific basins and between the mixed-layer and subsurface-layer water, which control the ENSO oscillation in the theoretical coupled model. By computing the CNOPs of the climatological basic state corresponding to the 1961–1975 (1981–1995) epoch, Duan and Mu [7] reproduced the observed decadal change of ENSO asymmetry qualitatively (Figure 9). On the basis of the physics described by the model, they [7] further explored the mechanism of ENSO amplitude asymmetry change on interdecadal scale. The results showed that the decadal change of ENSO amplitude asymmetry is induced by the change of nonlinear temperature advection, which is closely related to the decadal change of the tropical background state. Therefore, Duan and Mu [7] demonstrated that the decadal change of ENSO amplitude asymmetry results from the collective effect of the changes of the tropical background state and the nonlinearity. This finding in this study also suggest that the nonlinearity can explain not only the asymmetry of interannual ENSO, but also that of interdecadal ENSO, which may present a powerful evidence to the ENSO chaotic theory.

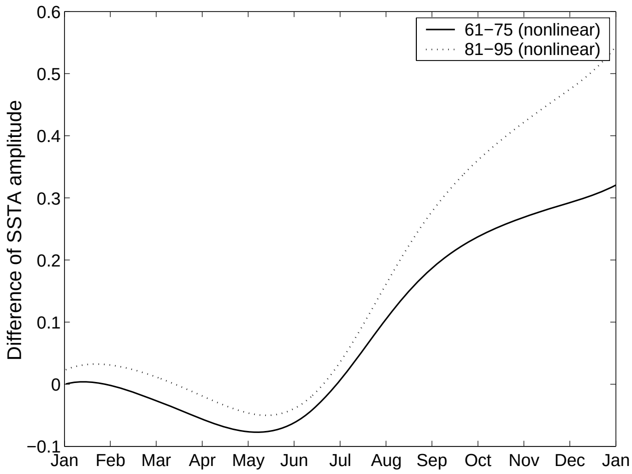

Figure 9. Amplitude differences between El Nino and La Nina in the adopted theoretical model, where the amplitude is measured by the absolute of SSTA. The solid (dot) line denotes the amplitude differences between El Nino and La Nina for 1961-1975 (1981-1995) basic state. The amplitude asymmetry of El Nino and La Nina for 1981-1995 basic state is larger than that for 1961-1975 basic state. This figure is from [7].

For the different types of nonlinearities in the ZC model, Duan et al. [8] investigated respectively their roles in ENSO amplitude asymmetry and identified clearly the origin of ENSO asymmetry, then emphasizing the decisive role of the nonlinear temperature advection. The nonlinear temperature advection enhances the El Nino amplitude but has little effect on La Nina, resulting in the asymmetry of ENSO in amplitude. They also demonstrated that the stronger the El Nino event is, the larger the nonlinear effect related to the NTA is and the more significant the ENSO asymmetry is, which can be used to explain the strong asymmetry of ENSO after 1976 and support the results of [7].

Other applications of CNOPs

Ensemble forecast is one of the major implementations for numerical weather and climate prediction. At European Centre for Medium-Range Weather Forecasts (ECMWF), LSVs have been utilized to generate the initial perturbations for its ensemble prediction system. Considering that CNOP is a natural extension of LSV into the nonlinear regime, it is reasonable to explore the possibility of using CNOPs to construct the initial perturbations. In Mu and Jiang [21], CNOPs is applied to ensemble prediction study by using a quasi-geostrophic model under the perfect model assumption. LSVs and CNOPs have been utilized to generate the initial perturbations for ensemble prediction experiments. The results are compared for forecast lengths of up to 14 days. It's found that the forecast skill of samples in which the first LSV is replaced by CNOP is comparatively better than that composed of only SVs in the medium range (day 6-day 14). Similarity index and Empirical orthogonal function (EOF) analysis are performed to explain the above numerical results.

Considering that LSVs have also been utilized in the adaptive observation research, particularly in the determinations of sensitivity area, Mu et al. [23] studied the applicability of CNOP in the determination of sensitivity area in adaptive observation. MM5 (Mesoscale Model 5) model and its adjoint form are utilized to study the effects of initial errors on the forecasts of two precipitation cases in July 2003 and in August 1996. The authors compare the differences between the structures of the CNOPs and the first linear singular vector (FSV), and calculate the developments of their total energies. It is found that the structures of CNOPs differ much from those of FSVs as well as the developments of their total energies. The results of sensitivity experiments indicate that the forecast results are more sensitive to the CNOP-type initial errors than the FSV-type ones. This indicates that it is feasible to use CNOP for the determination of sensitivity area in adaptive observation.

In addition, CNOP is also used to the studies of blocking, which is a typical large-scale circulation in the atmosphere with a characteristic time scale larger than that of synoptic motions. Blocking has long been recognized to have a profound effect upon regional weather and climate. On the other hand weather forecasts during periods when the atmospheric flow changes from strong zonal flow to blocking or vice versa frequently suffer from a rapid loss of predictability, which may be mainly due to the sensitivity of block onset and breakdown to initial perturbations. Thereby, the determination of initial perturbations for blocking onset will deepen our understanding of the low-frequency variability of the atmosphere and perhaps accordingly increase our skill in medium-range weather forecasting [15].

Mu and Jiang [22] studied the following problem: assuming that we have some information on the initial perturbations, e.g. they belong to an ensemble, which consists of the perturbations whose magnitudes are less than a given value, how to find out the perturbations belonging to this ensemble and triggering the blocking onset. By using a T21L3 quasigeostrophic (QG) model they reported that in some cases for the given initial ensemble perturbations, when CNOP triggers a transition to a blocking regime, whereas, LSV may not generate such a transition, which shows that nonlinear advection processes are fundamental for studying the weather regime transitions from zonal flow to blocking in the medium range. By choosing two objective functions and investigating the resulted CNOPs, they found that CNOPs obtained from the objective function of blocking-index form may trigger a transition to a blocking regime under some circumstances, whereas the CNOP related to the streamfunction squared norm fails to yield such transition. This demonstrates the importance of choosing a proper objective function when aiming at finding the perturbation yielding such transition. Besides, they also investigated the mechanism of perturbations of CNOP type trigging blocking onset.

For a grassland ecosystem model, Mu and Wang [24] studied the sensitivity and instability of the grassland ecosystem to finite-amplitude perturbations by CNOP approach. They found that linearly stable grassland (desert) states can be nonlinearly unstable with finite amplitude perturbations, which represent the human activities and (or) natural factors on the ecosystem. When the moisture index is between the two bifurcation points, a large enough finite amplitude perturbation can induce a transition from the grassland (desert) state to the desert (grassland) state. The thresholds of such transition along the bifurcation diagram of the moisture index are also given by the CNOP approach. The results also support the viewpoint of [38], which emphasis the shading effect of wilted biomass on the grassland ecosystem.

Main characteristics of CNOP and algorithms

CNOP is a natural generalization of LSV into the nonlinear category. Naturally it is of necessity to compare CNOPs with LSVs. Besides, it is very difficult to obtain CNOPs analytically. Keep these points in mind, the authors and their colleagues investigated the well-known models in atmospheric dynamics, looked for the numerical solution of CNOPs, compared the main characteristics of CNOPs with those of LSVs, and investigated the efficiency of numerical algorithms.

The CNOPs of simple models of ENSO and THC demonstrated that they bear three main characteristics. First, when linear approximations are not valid, there exist considerable differences between CNOPs and LSVs, which are characterized by two facts: one is that as initial perturbations, their spacial patters are quite different; the other is that there exist considerable differences between their linear and nonlinear evolutions at the time we are interested. Second, in some cases, there exist local CNOPs possessing clear physical meanings. Third, the numerical results show that CNOPs and local CNOPs all locate at the boundary of the domain prescribed by the constraint conditions in the phase space.

Mu and Zhang [25] has calculated the CNOPs of a two dimensional quasi-geotropic model with dimensions 512 by using the Sequential Quadratic Programming (SQP) algorithm. Their results verify the above properties found by using simple models. The SQP algorithm [3] used in [25] calculates the least value of a function of several variables subject to equality and inequality constraints. The experiments made by them show that the SQP solver is capable of obtaining CNOPs with problems of dimensions around 10^3 with nonlinear constraints.

Jiang et al [10] further employed a T21L3 quasi-geostrophic model and investigated its CNOPs. Their results not only verify the above mentioned three properties of CNOP , but also reveal that CNOP depends on the norm chosen; the streamfunction squared norm yields small-scale disturbances, the results of total energy norm is characterized by intermediate-scale disturbances, and in case of enstrophy norm, CNOP is typified by large-scale disturbance with large zonal flow contribution. The optimization algorithm employed to solve the optimization problem of dimension around 3×10^3 is Spectral Projected Gradient 2 (SPG2) [2], which calculates the least value of a function of several variables subject to box or ball constraints.

In the works on the predictability study of ENSO by using Zebiak and Cane model [37], and on the adaptive observation with MM5 model [23], the dimensions of nonlinear optimization variables are about 2×10^3 and 2×10^5 respectively. The algorithms they used are SPG2 too. These works show that the CNOPs all possess the mentioned three characteristics, and SPG2 method is able to solve problems with higher dimensions.

According to our numerical results, CNOPs or local CNOPs all located in the boundaries defined by the constrain conditions in the phase space. Recently Liu [12] proved theoretically that for any finite-dimensional dynamical system, CNOPs has such properties. Based on these results, Sun and Mu [29] reduced the calculations of CNOPs from solving constraint nonlinear optimizations to dealing with an unconstraint one, and compared the efficiencies of different algorithms, among which they found that Limited memory Broyden-Fletcher-Goldfarb-Shanno approach is the optimal one.

Discussion

This paper reviews the CNOP approach proposed by the authors and its applications to the sensitivity, stability, and predictability studies in GFD. Most of them are published, and some are in the review processes. It is clear from these studies that in the applications of CNOP, the corresponding cost function and the constraint conditions are of central importance, whose constructions should be capable of attacking the core of the physical problems that will be addressed. In the studies of ENSO predictability, to describe the evolution of the initial anomaly or the initial error, the nonlinear evolution of the perturbation measured by the norm of the state variables or the module of a state variable was chosen as the cost function. The obtained results showed its effectiveness. As to the constraint condition, the authors simply express it as belonging to a ball with the chosen norm. Obviously, the situation that initial perturbations belong to other kind of functional set can also be investigated. Furthermore, the constraint condition could be some physical law that initial perturbations should satisfy.

In the calculation of CNOPs, efficient nonlinear optimization algorithms are also essential, which guarantee the success of gaining CNOP. Atmospheric or oceanic flow motions are generally governed by intricate nonlinear models, which often have quite high dimensions. The involved nonlinear optimization problems could be difficult. Even in some cases, the problems are non-smooth. Nevertheless, encouraged by our works and the researches on the algorithms of [12, 29], it is expected that CNOP can be applied to realistic models with quite high dimensions and CNOP can also used to investigate the problems in other fields of fluid mechanics.

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